Secondary-Market Capital Prices and Financial Frictions In Business Cycles

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Abstract

Secondary market capital prices are procyclical and volatile. This paper quantifies the role of financial frictions in amplifying secondary market capital price changes in response to aggregate productivity and credit shocks. To do so, I develop a quantitative industry equilibrium model of investment and capital reallocation with heterogeneous firms, endogenous entry and exit, and heterogeneous capital durability. I calibrate the model to match key moments in the dynamics of the universe of U.S. firms. Financially constrained firms are net buyers of less durable capital because of its lower upfront cost despite its lower future resale value. Financially constrained firms are more responsive to aggregate productivity and credit shocks than less financially constrained firms, so financial frictions amplify movements in secondary market capital prices. I decompose secondary market capital price volatility into 90 percent fundamental volatility and 10 percent amplification from financial frictions. When secondary market capital prices are held fixed, aggregate output volatility is 111 percent of the baseline. Allowing secondary market capital prices to follow frictionless dynamics dampens output volatility by 8 percent and the remaining 3 percent stems from the amplified secondary market capital prices from financial frictions.

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1 Introduction

This paper is motivated by four facts. First, physical capital (e.g., equipment) is long-lived and actively traded in secondary markets. Among U.S. publicly traded firms, capital reallocation is about 25 percent of total investment (Eisfeldt and Rampini 2006). Second, physical capital is not only a factor of production but also serves as collateral. According to a Federal Reserve Board survey on small business lending, 88 percent of small business loans are secured. Third, young firms tend to use older physical capital (Ma, Murfin, and Pratt 2020). In my sample of tractors and semi-trucks, start-ups tend to use 3– to 4-year-old machines, while the average firm uses 1-year-old machines. Ma, Murfin, and Pratt (2020) and Lanteri and Rampini (2023) explore how this inverse relationship between firm age and machine age is driven by younger firms facing tighter financial frictions, empirically and theoretically. Fourth, prices of used physical capital are procyclical and volatile. For example, the correlation between the cyclical component of used vehicle prices and that of real industrial production is 23 percent, which is almost four times higher than prices for new vehicles. The volatility of the used price cycle is 4.84 percent, which is again about four times higher than the volatility of the new vehicle price cycle.

Taking these facts together, I ask what role do financial frictions play in amplifying secondary market capital prices? To answer this question, I build on the model of capital reallocation among heterogeneous firms developed by Lanteri and Rampini (2023). A key feature of the model is that there are two types of capital. New capital is elastically supplied and depreciates into old capital, whose price is determined by market clearing. Financially constrained firms effectively have higher discount rates, so they tilt their investment toward old capital to trade off its lower upfront cost against its lower resale value.

To this model, I add endogenous entry and exit, as entry is significantly procyclical and exit is significantly countercyclical (Lee and Mukoyama 2015), and both margins are important for aggregate dynamics (Clementi and Palazzo 2016). I endogenize the firm exit process, like Hopenhayn and Rogerson (1993), through the interaction of fixed operating costs and idiosyncratic productivity heterogeneity. With a fixed operating cost, firms exit when their productivity falls below a threshold because the fixed cost makes it unprofitable to continue operating. Further, because of financial frictions, the productivity exit threshold is decreasing in the firms' internal funds, so surviving firms accumulate internal funds and outgrow their financial frictions. I discipline the fixed operating cost using exit rates for the universe of U.S. firms measured by the Census Business Dynamics Statistics (BDS). This fixed cost approach contrasts with Lanteri and Rampini (2023) who model the firm life-cycle by assuming all firms face an identical, exogenous probability of exit each period, regardless of the firms' characteristics.

I then study business cycle dynamics arising from aggregate total factor productivity (TFP) shocks and aggregate credit shocks using methods from Boppart, Krusell, and Mitman (2018). I estimate the dynamics of these shocks to match the persistence, volatility, and correlation of real industrial production and real aggregate nonfinancial noncorporate debt.

I find that financial amplification constitutes 10 percent of the volatility of secondary market physical capital prices. Specifically, I compare price volatility in my calibrated baseline model to that in a frictionless benchmark. In the frictionless benchmark, price volatility is 90 percent of that in the baseline model, implying that the remaining 10 percent is attributable to amplification from financial frictions.

With this result in hand, I quantify the cyclical dynamics of secondary market capital prices in amplifying or dampening aggregate output and aggregate investment volatility. I decompose the volatility of output into three components: (1) the volatility of aggregate output when secondary market capital prices are held fixed at their steady-state level, (2) when secondary market capital prices follow the dynamics of the (less volatile) frictionless benchmark, and (3) the remaining volatility in the baseline model. I find that when secondary market capital prices are held fixed, the model generates 111 percent of the baseline aggregate output volatility. This suggests that secondary market capital markets dampen aggregate productivity shocks. When secondary market capital prices follow the dynamics in the frictionless benchmark, 8 percent of the excess aggregate output volatility is dampened. The remaining 3 percent of the aggregate output volatility is generated by the amplified dynamics of secondary market capital prices from the financial frictions.

The rest of this paper is organized as follows. Section 2 reviews related literature. Section 3 presents the empirical motivating facts. Section 4 lays out the illustrative model model with overlapping generations of firms. Section 5 lays out the quantitative model with long-lived firms. Section 6 discusses computation and calibration. Section 7 evaluates the model by the movements of aggregates across the business cycle. Section 8 presents the results, including the effects of financial frictions on secondary market capital prices, the effects of secondary market capital prices on output, and optimal policy. Section 9 concludes.

2 Related Literature

This paper relates to a large literature on capital reallocation, beginning with Ramey and Shapiro (2001), who document the closure of an aircraft manufacturing plant following reduced government spending on military aircraft with the end of the Cold War. Eisfeldt and Shi (2018) provide a comprehensive review. Eisfeldt and Rampini (2006) show that, while capital reallocation is procyclical, the benefits of reallocation appear countercyclical, suggesting that the costs associated with capital reallocation are also countercyclical. Li and Whited (2015) and Fuchs, Green, and Papnikolaou (2016) examine the role of information asymmetry and adverse selection in used physical capital markets. Gavazza (2010, 2011a, 2011b, 2016), Wright, Xiao, and Zhu (2018, 2020), Cui, Wright, and Zhu (2025), and Ottonello (2014) focus on the role of search in secondary capital markets. Lanteri (2018) emphasizes the role of capital specificity. My quantitative findings build on Lanteri and Rampini (2023) who examine a stationary environment with heterogeneous firms, capital vintages, and financial frictions.

This paper also connects to a large literature on capital misallocation driven by financial frictions, including the work of Restuccia and Rogerson (2008), Hsieh and Klenow (2009), Beura, Kaboski, and Shin (2011), Midrigan and Xu (2014), Moll (2014), and David and Venkateswaran (2019). Restuccia and Rogerson (2008) highlight the role of financial frictions in causing capital misallocation and lowering measured TFP. Hsieh and Klenow (2009) document substantial dispersion in the average revenue products of capital and labor in China and India relative to the U.S., finding that such misallocation could explain 30-50 percent of the cross-country differences in measured TFP. David and Venkateswaran (2019) decompose several sources of capital misallocation and identify capital adjustment, not financial frictions, as the most relevant driver of misallocation for publicly traded U.S. firms.

This paper also relates to the literature about capital heterogeneity. Closest are Rampini (2019) and Lanteri and Rampini (2023), which focus on how capital defers by its durability. In this view, more durable capital is easier to finance, but is more also expensive in general equilibrium. Jovanovic and Yatsenko (2012) surveys how capital defers by its embodied technology.

3 Motivating Facts

In the introduction, I highlight four motivating facts. The first is that physical capital (e.g., equipment) is long-lived and actively traded in secondary markets as highlighted by Eisfeldt and Rampini (2006).[1] The second is that physical capital is not only a factor of production, but it also serves as collateral as highlighted by the Federal Reserve Board survey on small business lending.[2,3] In this section, I delve more into the last two motivating facts mentioned in the introduction.

3.1 Secondary market capital prices are procyclical and volatile

Figure 2 shows the cyclical component of new and used vehicle prices. Lanteri (2018) computes that vehicles are roughly 12 percent of the U.S. equipment stock. The price data comes from the components of the Consumer Price Index. The "new" and "used" lines are the new and used vehicles in U.S. urban average, respectively, divided by the headline consumer price index then averaged to a quarterly frequency and Hodrick–Prescott filtered with a smoothing parameter of 1,600. Table 1 shows that the correlation of real GDP and used prices is twice the correlation of real GDP and new prices. Further, used prices are four times as volatile as new prices. The table shows that used prices are procyclical and volatility both relative

[1] Sales of property, plants, and equipment and acquisitions, relative to capital expenditures and acquisitions among Compustat firms.

[2] Fraction of loans under $1 million secured by collateral in 2016. Survey of Terms of Business Lending E.2 by the Federal Reserve Board. Also highlighted by Gopal (2023).

[3] The importance of secured debt for private firms is in contrast to public firms. Eighty percent of public firms have earnings-based borrowing constraints instead of collateral-based borrowing constraints (Li and Mian, 2020) and there has been a secular decline in secured corporate bonds, leaving only 11 percent of corporate bonds secured as of 2019 (Benmelech, Kumar, and Rajan, 2024).

to the general price level or relative to new vehicles. Lanteri (2018) shows similar business cycle patterns for prices of aircraft, ships, and construction equipment.

Figure 1: Cyclical Component of New and Used Vehicle Prices

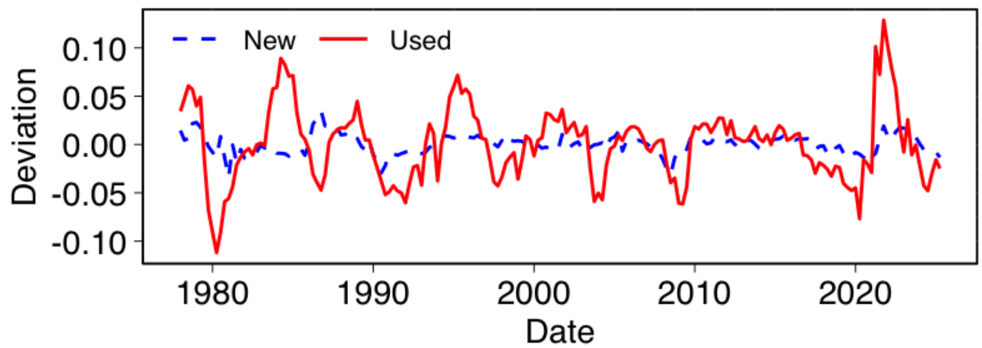


Table 1: Business Cycle Properties of New and Used Vehicle Prices

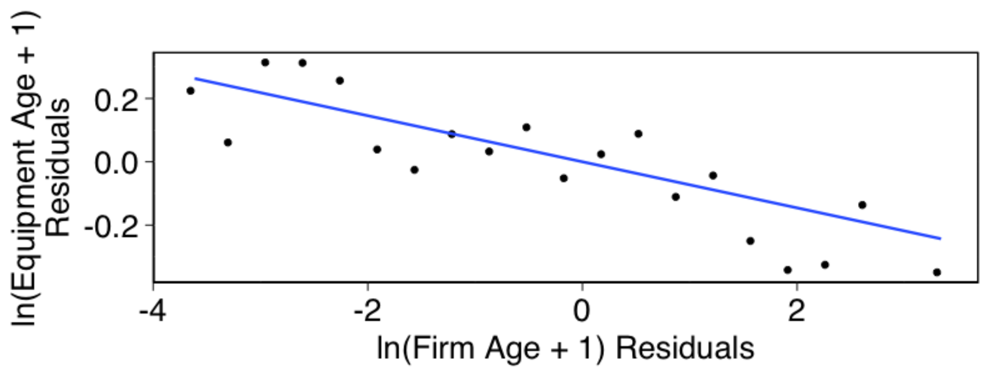
%	$\frac{\text{New}}{\text{CPI}}$	$\frac{\text{Old}}{\text{CPI}}$	$\frac{\text{Old}}{\text{New}}$
Volatility	1.23	4.84	4.46
Output Correlation	6.35	23.08	23.29
Autocorrelation	84.25	88.00	86.36

3.2 Younger firms use older capital

To show that younger firms use older capital, I collect Uniform Commercial Code (UCC) filings from California, Colorado, Connecticut, Florida, and Virginia. When borrowing is collateralized by a debtor's assets, lenders can file UCC financing statements with the Secretaries of State in the state that the debtor is incorporated. These filings both serves to perfect the lender's security interest in the event of bankruptcy or default and publicly indicate the lender's security interest in a debtor's assets to other lenders. I bulk download data that includes name and address of the debtor, name and address of lender, and whether the filing is initial or amendment. I then merge the UCC filings with state-level business registry to get the data of incorporation. I scrape and parse the collateral information from PDFs filings. I use 17-digit serial number of movable capital, like tractors, excavator, semi-trucks, and truck-trailers, because the structure of the serial number assures excellent data quality and embedded in the serial number is the model year. Appendix A details the data collection process.

Figure 2 shows a binned scatterplot of firm age, which is the difference between the filing date and the date of incorporation, versus machine age, which is the filing year minus the model year. Fixed effects for lender type, filing year, and state are included. There is a significant downward sloping relationship. This relationship between corresponds to a start-up using machines that are 3-4 years old on average while an average firm uses machines that are 1 year old.

Figure 2: Relationship between Firm Age and Machine Age



Ma, Murfin, and Pratt (2020) use Uniform Commercial Code filings to show that younger firms tend to use older machines across many states and types of machines.

4 Illustrative Model

This section outlines an illustrative overlapping generations model of how financially constrained firms can amplify the demand for used capital in response to aggregate shocks. There is a common aggregate shock $\varepsilon_t \sim \mathcal{N}(0, 1)$ that has four exogenous effects. A positive ε_t raises aggregate productivity a_t as in canonical real business cycle models like in Kydland and Prescott (1982). A positive ε_t also raises the extent to which firms can borrow against their capital θ_t like in Khan and Thomas (2013). A positive ε_t also increases the mass of both entrants μ_t and incumbents υ_t to capture procyclical firm entry and countercyclical firm exit as documented by Campbell (1998). In this section, I consider a single shock with four exogenous effects for tractability. In the quantitative model outlined in Section 5, I consider imperfectly correlated shocks to aggregate productivity and credit conditions where its correlation is calibrated to match the comovement of aggregate output and aggregate borrowing, while firm entry and firm exit respond endogenously.

Production is decreasing-returns-to-scale of constant elasticity of substitution aggregator of new and used capital

$$
\begin{aligned}
f(k_N, k_O; a) &= a[g(k_N, k_O)]^{\alpha} \\
g(k_N, k_O) &= \left[\nu^{\frac{1}{\epsilon}} k_N^{\frac{\epsilon-1}{\epsilon}} + (1-\nu)^{\frac{1}{\epsilon}} k_O^{\frac{\epsilon-1}{\epsilon}}\right]^{\frac{\epsilon}{\epsilon-1}}
\end{aligned}
$$

where $\alpha < 1$ captures span of control, $\epsilon > 0$ is the elasticity of substitution between new and used capital, and ν is the weight on new capital. As in Kydland and Prescott (1982), ε_t affects aggregate productivity. In particular, aggregate productivity is an AR(1) process $\log(a_t) = \rho_a \log(a_{t-1}) + \sigma_a \varepsilon_t$ where $\rho_a \in (0, 1)$ and $\sigma_a > 0$. As in Lanteri and Rampini (2023), when k_N units of new capital is used in production, it depreciates into $(1-\delta)k_N$ units of new capital and δk_N units of used capital. When k_O units of used capital are used in production, it depreciates into $(1-\delta)k_O$ units of used capital and δk_O units are rendered inoperable. New capital is elasticity supplied at a unit price and used capital has an endogenous price q_t that is pinned down by market clearing.

Firms live for three periods. In the first period, a firm of type j starts with an exogenous level of net worth w^j. The firm invests in new and used capital funded by its net worth as well as external equity that requires an extra return over internal funds as in Gomes (2001) and borrowing collateralized by the market-value of capital investment as in Kiyotaki and Moore (1994). In its second period, the firm uses its capital to produces depreciating the capital, and repays its debt. Then the firm invests in next period's new and used capital, borrows, and either pays dividends or issues more external equity. In its third and final period, the firm produces and it sells its depreciated capital, and repays its debts.

There are two types of firms that are heterogeneous in exogenous net worth w^j at birth. Mass μ_t of "entrants" have no initial net worth $w^E = 0$. I use superscript E to denote entrant allocations. The aggregate shock exogenously affects the mass of entrants $\mu_t = \mu \exp(\chi_t)$

with $\chi_t = \rho_\chi \chi_{t-1} + \sigma_\chi \varepsilon_t$ where $\rho_\chi \in (0,1)$ and $\sigma_\chi > 0$. In the quantitative model outlined in Section 5, I endogenize the effect of aggregate productivity and credit shocks on the mass of entrant by modeling an endogenous entry decision of potential entrants. There is also mass υ_t of "incumbents", whose allocations are denoted with superscript I. Incumbents have net worth $w^I \geq \bar{w}$ where $\bar{w}$ is the amount of funds requires to invest in capital at the optimal level based on the decreasing-returns-to-scale production for both periods. Without loss of generality, I treat $\bar{w}$ as arbitrarily large. The aggregate shock also exogenously affects the mass of incumbents $\upsilon_t = \upsilon \exp(\eta_t)$ with $\eta_t = \rho_\eta \eta_{t-1} + \sigma_\eta \varepsilon_t$. The idea here is that firm exit is countercyclical, so in a boom, when firm exit drops, the mass of incumbents grows. Like with entry, in the quantitative model outlined in Section 5, I endogenize firm exit through incorporating a fixed operating cost that prompts unproductive and low net worth firms to exit. Capturing firm exit as the increase in the mass of incumbents without capturing this selection is akin to a model where firms exit as in Lanteri and Rampini (2023). I normalize the mass of producing firms, that is middle-aged and old, in steady-state to a unit mass, so $2\mu + 2\upsilon = 1$. With two types by net worth and three ages, there are six overall types of firms in each period.

There are two financial frictions. First, equity issuance is costly like Gomes (2001). A negative dividend incurs an additional cost $\phi(-d)$ on households where ϕ is increasing $\phi'(-d) > 0$ and strictly convex $\phi''(-d) > 0$. The firm internalizes this additional cost despite it not directly paying the cost. In particular, I use a power function of $-d$

$$\phi(-d) = \phi_0 \mathbb{1}\{d < 0\} d^{\phi_1}$$

with $\phi_0 > 0$ and $\phi_1 > 1$.

Second, firms can only borrow up to a fraction θ_t of the current market value of their capital investment like Kiyotaki and Moore (1994):

$$b^{j,h}_{t+1} \leq \theta_t (k^{j,h}_{N,t+1} + q_t k^{j,h}_{O,t+1})$$

for firm age $h = \{1, 2\}$. The microfoundation for this financial friction is that there is a limited enforcement of debt contracts and a lender can destroy $1 - \theta_t$ fraction of a firms capital if it defaults but cannot exclude the firm from credit markets. I use the current price of used capital because the future market-value of capital depend on the future used capital price, which is a random variable. Thus, since shocks to productivity are unbounded, the future price is unbounded. If the inequality must hold for all future periods, then the solution is trivial with $b_{t+1}^{j,h} = 0$. This removes any interesting interaction between capital prices and borrowing capacity. Bianchi and Mendoza (2018) also use the current price in a similar constraint. I leave the exploration of defaultable debt like Hennessy and Whited (2005) where bankruptcy recoveries depend on future used capital prices for future work. The aggregate shock also affects collateralizability $\theta_t = \theta \exp(\omega_t)$ where $\omega_t = \rho_\omega \omega_{t-1} + \sigma_\omega \varepsilon_t$ where $\rho_\omega \in (0,1)$ and $\sigma_\omega > 0$. In the quantitative model, I explore imperfectly correlated productivity and credit shocks.

The problem of young firm with net worth w in period t is to choose new and used capital, borrowing, and dividends:

$$
\begin{aligned}
V_t^0(w^j) = & \max_{k_{N,t+1}^{j,1}, k_{O,t+1}^{j,1}, b_{t+1}^{j,1}, d_t^{j,0}} d_t^{j,0} - \phi(-d_t^{j,0}) + \frac{1}{R}\mathbb{E}_t[V_{t+1}^1(k_{N,t+1}^{j,1}, k_{O,t+1}^{j,1}, b_{t+1}^{j,1})] \\
\text{s.t. } & d_t^{j,0} = w - k_{N,t+1}^{j,1} - q_t k_{O,t+1}^{j,1} + b_{t+1}^{j,1} \\
& b_{t+1}^{j,1} \leq \theta_t(k_{N,t+1}^{j,1} + q_t k_{O,t+1}^{j,1})
\end{aligned}
$$

The problem of middle-aged firm with new capital $k_{N,t+1}^{j,2}$, used capital $k_{O,t+1}^{j,2}$, and borrowing $b_{t+1}^{j,2}$ is to choose new and used capital, borrowing, and dividends:

$$
\begin{aligned}
V_{t+1}^1(k_{N,t+1}^{j,1}, k_{O,t+1}^{j,1}, b_{t+1}^{j,1}) = & \max_{k_{N,t+2}^{j,2}, k_{O,t+2}^{j,2}, b_{t+2}^{j,2}, d_{t+1}^{j,1}} d_{t+1}^{j,1} - \phi(-d_{t+1}^{j,1}) + \frac{1}{R}\mathbb{E}_{t+1}[V_{t+2}^2(k_{N,t+2}^{j,2}, k_{O,t+2}^{j,2}, b_{t+2}^{j,2})] \\
\text{s.t. } d_{t+1}^{j,1} = & f(k_{N,t+1}^{j,1}, k_{O,t+1}^{j,1}; a_{t+1}) + (1-\delta)k_{N,t+1}^{j,1} + q_{t+1}[\delta k_{N,t+1}^{j,1} + (1-\delta)k_{O,t+1}^{j,1}] - Rb_{t+1}^{j,1} \\
& - k_{N,t+2}^{j,2} - q_{t+1}k_{O,t+2}^{j,2} + b_{t+2}^{j,2} \\
b_{t+2}^{j,2} \leq & \theta_{t+1}(k_{N,t+2}^{j,2} + q_{t+1}k_{O,t+2}^{j,2})
\end{aligned}
$$

The value of an used firm with new capital $k^{j,2}_{N,t+2}$, used capital $k^{j,2}_{O,t+2}$, and borrowing $b^{j,2}_{t+2}$:

$$U^2_{t+2}(k^{j,2}_{N,t+2}, k^{j,2}_{O,t+2}, b^{j,2}_{t+2}) = d^{j,2}_{t+2} - \phi(-d^{j,2}_{t+2})$$
$$\text{where } d^{j,2}_{t+2} = f(k^{j,2}_{N,t+2}, k^{j,2}_{O,t+2}; a_{t+2}) + (1-\delta)k^{j,2}_{N,t+2} + q_{t+2}[\delta k^{j,2}_{N,t+2} + (1-\delta)k^{j,2}_{O,t+2}] - Rb^{j,2}_{t+2}$$

The aggregate quantaties in the model are

$$K_{O,t} \equiv \mu_{t-1}k^{E,1}_{O,t} + \mu_{t-2}k^{E,2}_{O,t} + \upsilon_{t-1}k^{I,1}_{O,t} + \upsilon_{t-2}k^{I,2}_{O,t}$$
$$K_{N,t} \equiv \mu_{t-1}k^{E,1}_{N,t} + \mu_{t-2}k^{E,2}_{N,t} + \upsilon_{t-1}k^{I,1}_{N,t} + \upsilon_{t-2}k^{I,2}_{N,t}$$

Clearing in used capital market requires that

$$K_{O,t+1} = \delta K_{N,t} + (1-\delta)K_{O,t}$$

An equilibrium is a set of allocations and used capital prices where the allocations maximize firm problems and markets clear.

I solve the firm problem using backward induction in Appendix B. The solution is characterized by the following optimality conditions

$$1 = \mathbb{E}_t\left[\frac{1+\phi'(-d^{j,h+1}_{t+1})}{R[1+\phi'(-d^{j,h}_t) - \lambda^{j,h}_t\theta_t]}[f_1(k^{j,h+1}_{N,t+1}, k^{j,h+1}_{O,t+1}; a_{t+1}) + 1 - \delta + q_{t+1}\delta]\right] \tag{1}$$
$$q_t = \mathbb{E}_t\left[\frac{1+\phi'(-d^{h+1}_{t+1})}{R[1+\phi'(-d^{h}_t) - \lambda^{j,h}_t\theta_t]}[f_2(k^{h+1}_{N,t+1}, k^{h+1}_{O,t+1}; a_{t+1}) + (1-\delta)q_{t+1}]\right] \tag{2}$$
$$\lambda^{j,h}_t = \phi'(-d^{j,h}_t) - \mathbb{E}_t[\phi'(-d^{j,h+1}_{t+1})] \tag{3}$$

for firm ages $h \in \{0,1\}$ where $\lambda^{j,h}_t$ is the multiplier on the collateral constraint for firm with age h and initial net worth w_j in period t. Equation 1 is the investment Euler equation for new capital. The left-hand side is the marginal cost of new capital, which is normalized to one. The right-hand side is marginal benefit adjusted for financial constraints. The marginal benefit is the marginal product of capital and resale value. For a unit of new capital, $1-\delta$ of new capital is remaining and can be resold at unit price as well as δ units of used capital that can be sold at q_{t+1}. The presence of financial frictions are captured in the first term. Equation 3 is the first order condition with respect to borrowing. The value of borrowing today is to equalize today's marginal equity issuance cost and tomorrow's expected marginal equity issuance cost. If they are equalized, then it wouldn't be optimal for the firm to borrow any further, so the shadow value of the collateral constraint is zero. Financially constrained firms are effectively impatient with higher effective discount factors than unconstrained firms. Equation 2 is the investment Euler equation for used capital. The left-hand side is the marginal cost of q_t and the right-hand side is the expected marginal benefit, including the marginal product of capital plus the resale value. As discussed by Lanteri and Rampini (2023), more financially constrained firms tilt investment toward used capital. With higher discount factors, financially constrained firms discount more the resale value of new capital, which is the q_{t+1} on the right-hand side of equation 1, but do not discount the lower upfront cost of used capital, which is the q_t on the left-hand isde of equation 2.

Incumbents have sufficient funds to invest at the optimal level without issuing any equity, $d_t^{I,0} \geq 0$. In particular, they have enough that they can save through $b_t^{I,1} < 0$ when they are young, so they don't need to issue equity when middle-aged and used, $d_t^{I,1}, d_t^{I,2} \geq 0$. Since they do not incur equity issuance costs $\phi'(d_t^{I,1}) = \phi'(d_t^{I,2}) = \phi'(d_t^{I,3}) = 0$, the shadow value of their collateral constraints is likewise zero $\lambda_t^{I,0} = \lambda_t^{I,1} = 0$. Thus, their investment Euler equations lose any adjust in effective discount factor associated with financial constraints

$$
\begin{aligned}
1 &= \frac{1}{R}\mathbb{E}_t[f_1(k_{N,t+1}^{I,h+1}, k_{O,t+1}^{I,h+1}; a_{t+1}) + 1 - \delta + q_{t+1}\delta] \\
q_t &= \frac{1}{R}\mathbb{E}_t[f_2(k_{N,t+1}^{I,h+1}, k_{O,t+1}^{I,h+1}; a_{t+1}) + q_{t+1}(1-\delta)]
\end{aligned}
$$

Thus, young and middle-aged incumbents choose the same investment $k_{N,t}^{I,1} = k_{N,t}^{I,2} \equiv k_{N,t}^{I}, k_{O,t}^{I,1} = k_{O,t}^{I,2} \equiv k_{O,t}^{I}$. This simplifies aggregate new and used capital as follows.

$$
\begin{aligned}
K_{O,t} &= \mu_{t-1}k_{O,t}^{E,1} + \mu_{t-2}k_{O,t}^{E,2} + (\upsilon_{t-1} + \upsilon_{t-2})k_{O,t}^{I} \\
K_{N,t} &= \mu_{t-1}k_{N,t}^{E,1} + \mu_{t-2}k_{N,t}^{E,2} + (\upsilon_{t-1} + \upsilon_{t-2})k_{N,t}^{I}
\end{aligned}
$$

For incumbents, the Modigliani-Miller irrelevance theorem holds, so its choice of dividends versus borrowing is not determined.

For entrants, its young dividend is smaller than its expected dividend when middle-aged, $d^0_{E,t} < \mathbb{E}_t[d^{E,1}_{t+1}]$. Thus, its marginal equity issuance cost is also higher $\phi'(d^{E,0}_t) < \mathbb{E}_t[\phi'(d^{E,1}_{t+1})]$. Implying that entrants would more if it could $\lambda^{E,0}_t > 0$. Similarly, its middle-aged dividend is larger than its expected old-aged dividend $d^1_{E,t+1} < \mathbb{E}_{t+1}[d^{E,2}_{t+2}]$, implying that $\phi'(d^{E,1}_t) < \mathbb{E}_{t+1}[\phi'(d^{E,2}_{t+1})]$ and $\lambda^{E,1}_t > 0$. Since its collateral constraints is binding, borrowing is pinned down by

$$b^{E,h+1}_{t+1} = \theta_t(k^{E,h+1}_{N,t+1} + q_t k^{E,h+1}_{O,t+1})$$

for $h \in \{0, 1\}$.

With this characterization, I can now explore how financial frictions amplify demand for used capital in response to the aggregate shock. Consider long string of $\varepsilon_t = 0$, so exogeneous variables are at their steady-state levels including $a_t = 0$, $\theta_t = \theta$, $\mu_t = \mu$, $\upsilon_t = \upsilon$. Allocation are also at their steady-state levels, denote by the lack of time subscript, including $K_{N,t} = K_N$ and $K_{O,t} = K_O$. At $t = \tau$, there is a positive shock $\varepsilon_\tau > 0$. Market-clearing in τ is

$$\mu_\tau k^{E,1}_{O,\tau+1} + \mu k^{E,2}_{O,\tau+1} + (\upsilon_\tau + \upsilon) k^I_{O,\tau+1} = \delta K_N + (1-\delta) K_O \tag{4}$$

The demand for used capital, LHS of (4), increases through the following six channels.

First, all firms have higher optimal size with $\varepsilon_\tau > 0$. Consider the used capital investment Euler equation where only future expected productivity is affected by the aggregate shock

$$q \leq \mathbb{E}_\tau \left[\frac{1 + \phi'(-d^{j,h})}{R[1 + \phi'(-d^{j,h+1}) - \lambda^{j,h}\theta]} [f_2(k^{j,h}_N, k^{j,h}_O; a_{\tau+1}) + q(1-\delta)] \right]$$

Higher future expected productivity $a_{\tau+1}$ raises the marginal product of used capital $f_2(k^j_N, k^j_O; a_{\tau+1})$, so the benefit of investing in used capital outweighs the cost. All firms invest in more used capital, lowering their marginal product of used capital until the Euler equation holds. This put upward pressure on the price. This channel relies on the decreasing-returns-to-scale production and the irreversibility of used capital that is built into its deprecation structure. In particular, this channel would be present even in a representative firm version model, for example if the mass of entrants is zero, $\mu_t = 0$.

Second, middle-aged entrants have higher cashflows that ease their financial constraints and increase their investments including in used capital. To see this channel, consider the used capital investment Euler equation of middle-aged entrants where only current productivity changes:

$$q \leq \mathbb{E}_\tau \left[\frac{1 + \phi'(-d^{E,2})}{R[1 + \phi'(-\hat{d}_\tau^{E,1}) - \lambda^{E,1}\theta]} [f_2(k_N^{E,2}, k_O^{E,2}; 0) + q(1-\delta)] \right]$$
$$\hat{d}_\tau^{E,1} = f(k_N^{E,1}, k_O^{E,1}; a_\tau) + (1-\delta)k_N^{E,1} + q_\tau[\delta k_N^{E,1} + (1-\delta)k_O^{E,1}] - Rb^{E,1} - k_N^{E,2} - qk_O^{E,2} + b^{E,2}$$

Higher current productivity raises middle-aged entrant production $f(k_N^{E,1}, k_O^{E,1}; a_\tau)$. This increased cashflow increases their dividend holding future capital and borrowing choices fixed at the steady-state levels $\hat{d}_\tau^{E,1} > d^{E,1}$. This higher dividend lowers marginal equity issuance cost $\phi'(-\hat{d}_\tau^{E,1}) < \phi'(-d^{E,1})$. The lower marginal equity issuance raises the middle-aged entrants' effective discount factor, so the middle-aged entrant effectively more patient. The middle-aged entrant, thus, increases its used capital investment to lower the marginal product of used capital. This channel is strengthen since, as shown by Lanteri and Rampini (2023), constrained firms tilt their investment toward used capital. Notice that this channel requires financial constraints, so it would not be present in a model with an unconstrained representative firm. Neither would not be present in an overlapping generations model with firm that start with an exogenous net worth and live for only two periods.

Third, the aggregate shock $\varepsilon_\tau > 0$ exogenously raises collateralizability θ_τ, so young and middle-aged entrants can borrow more against their capital. To see this, consider the Euler equation for used capital where only collateralizability changes:

$$q \leq \mathbb{E}_\tau \left[\frac{1 + \phi'(-d^{E,h})}{R[1 + \phi'(-d^{E,h+1}) - \lambda^{E,h}\theta_\tau]} [f_2(k_N^{E,h}, k_O^{E,h}; 0) + q(1-\delta)] \right]$$

Higher collateralizability lowers the effective discount factor for entrants, so they choose higher used capital levels of used capital. Again, the fact that constrained firms tilt their investments toward used capital strengthens this channel. This channel requires financially-constrained firms and secured debt.

Fourth, there is a collateral feedback channel through used capital prices like Kiyotaki-Moore (1994). To see this, I rearrange the used capital Euler equation where only the used capital price that firms use to borrow against responds to the shock $\varepsilon_\tau > 0$ through some other channel $\hat{q}_\tau > q$:

$$q \leq \mathbb{E}_\tau \left[\frac{1 + \phi'(-d^{E,h})}{R[1 + \phi'(-d^{E,h+1})]} [f_2(k_N^{E,h}, k_O^{E,h}; 0) + q(1 - \delta)] \right] + \hat{q}_\tau \frac{\lambda^{E,h}\theta}{1 + \phi'(-d^{E,h+1})}$$

Since the market value of used capital is higher, so young and middle-aged entrants can borrow more against it. Again, the fact that constrained firms tilt their investments toward used capital strengthens this channel. Like the exogenous increase in collateralizability, this channel requires financially constrained firms and collateralized debt.

Fifth, the aggregate shock $\varepsilon_\tau > 0$ exogenously raises the mass of entrants μ_τ. To see this, consider market-clearing where only the mass of entrants responds to the shock.

$$\mu_\tau k_O^{E,1} + \mu k_O^{E,2} + (\upsilon + \upsilon) k_O^I > \delta K_N + (1 - \delta) K_O$$

This channel is strengthened by entrants tilting their investment toward used capital. Naturally, this channel requires entry to respond procyclically. Here, that response is mechanical, while in the quantitative model, the entry responses endogenously to changes in productivity and credit.

Sixth, the aggregate shock $\varepsilon_\tau > 0$ exogenously raises the mass of incumbents υ_τ. Again, consider market-clearing where only the mass of incumbents responds to the shock.

$$\mu k_O^{E,1} + \mu k_O^{E,2} + (\upsilon_\tau + \upsilon) k_O^I > \delta K_N + (1 - \delta) K_O$$

This channel requires that firm exit responds countercyclically and so the mass of firm responds procyclically. As with entrants, this response is mechanical, while in the quantitative model, firm exit is endogenous. Furthermore, since the quantitative model endogenizes firm exit with a fixed operating cost, there is selection induced by firm exit. Low net worth and low productivity firms exist. Since firms that exit are low net worth and financial constrained firms rely on used capital, an aggregate shock lowering exit disproportionately impacts firms that rely on used capital. This strengthens this channel. In this illustrative model, the aggregate shock exogenously changing the mass of entrant is akin to an exogenously exiting firm, as modeled by Lanteri and Rampini (2023).

5 Quantitative Model

I outline the model in this section. Time is discrete and infinite. Firms maximize the present-value of their dividends at constant interest rate R. Firms are heterogeneous in four dimensions $s_{i,t} \equiv (z_{i,t}, k_{N,i,t}, k_{O,i,t}, b_{i,t})$ for firm i operating at date t where $z_{i,t} \sim Q_z(z_{i,t}|z_{i,t-1})$ is idiosyncratic total factor productivity, $k_{N,i,t}$ is new capital, $k_{O,i,t}$ is old capital, and $b_{i,t}$ is noncontigent borrowing. There is an aggregate component to total factor productivity that is common to all firms a_t. Firms produce with constant elasticity of substitution aggregator of new and old capital nested in decreasing-returns-to-scale production

$$f(z, k_N, k_O; a) = \exp(z + a)\left[\nu^{\frac{1}{\epsilon}} k_N^{\frac{\epsilon-1}{\epsilon}} + (1-\nu)^{\frac{1}{\epsilon}} k_O^{\frac{\epsilon-1}{\epsilon}}\right]^{\frac{\alpha\epsilon}{\epsilon-1}}$$

where $\nu \in [0, 1]$ is the weight on new capital, $\epsilon \geq 0$ is the elasticity of substitution, and $\alpha \in (0, 1]$ captures the span of control. Through production, fraction $\delta_N \in (0, 1]$ of new capital depreciates into old and fraction $\delta_O \in (0, 1]$ old capital becomes worthless. New capital is elastically supplied at unit price and old capital is traded at market-clearing price $q_t > 0$.

Before investing in capital and taking on new debt, firms can choose whether to exit. Exiting firms pay their production and the market value of their depreciated capital net of their debt repayment as an exiting dividend. Continuing firms pay fixed operating cost c_F, choose next period's capital stocks ($k_{N,i,t+1}$,$k_{O,i,t+1}$), and new borrowing $b_{i,t+1}$. A unit mass of potential entrants draw heterogeneous productivity signals $z_{E,i} \sim Q_E$ and choose whether to setup. Entrants that setup firms pay a setup cost c_E and choose capital stocks and borrowing. The initial idiosyncratic productivity $z_{i,t+1}$ of entrants from period t with signal $z_{E,i}$ is drawn from distribution $Q_{E,z}(z_{i,t+1}|z_{E,i})$. When investing in capital and taking on new debt, continuing incumbent firms and entrant firms face two financial frictions. First, firms internalize that equity issuance is costly to shareholders, as in Gomes (2001),

$$\phi(-d) = \mathbb{1}\{d < 0\}\phi_0(-d)^2$$

where $\phi_0 \geq 0$ captures the scale of equity issuance costs. Second, as in Kiyotaki and Moore (1994), borrowing is constrained by a collateralizable fraction $\theta_t \geq 0$ of the market value of the firm's capital stock

$$b_{i,t+1} \leq \theta_t \Big[k_{N,i,t+1} + q_t k_{O,i,t+1} \Big].$$

The collateral constraint arises from microfoundations of a commitment friction or limited enforcement of debt contracts without exclusion where the lender can destroy a fraction of the borrowers capital if the borrower defaults. Using current prices, instead of future prices like Kiyotaki and Moore (1994), follows Bianchi and Mendoza (2018). In the spirit of Khan and Thomas (2013), I explore the effect of shocks to collateralizability parameterized as

$$\theta_t \equiv \bar{\theta} \exp(\omega_t).$$

where $\bar{\theta}$ is the baseline collateralizability and ω_t is a credit shock.

Within a period, events follow:

1. Idiosyncratic productivity is drawn and firms produce.
2. Incumbent firms choose whether to exit.
3. Potential entrant choose to whether to set up.
4. Firms invest in new and old capital, borrow debt, pay out dividends, and incur equity issuance costs.
5. Firms can choose to default.

Here, I formalize the problem of incumbent firms and entrants using Bellman equations.

At the beginning of date t, the incumbent firm first chooses whether to exit.

$$V_t(s) = \max\{V_t^X(s), V_t^C(s)\} \tag{5}$$

Use $x_t(s) \equiv 0$ to denote that firm s chooses to continue and $x_t(s) \equiv 1$ to denote that firm s chooses to exit. If a firm exits, it pays an exiting dividend of its net worth, which is its production and the market value of its capital after repaying its debt.

$$\begin{aligned} V_t^X(s) &= \pi_t(s) \\ &\equiv f(z, k_N, k_O; a_t) + (1-\delta_N)k_N + q_t[\delta_N k_N + (1-\delta_O)k_O] - Rb \end{aligned} \tag{6}$$

Continuing firms pay fixed operating cost, produce, and choose new capital, borrowing, and dividends subject to the collateral constraint.

$$V_t^C(s) = \max_{(d,b')\in\mathbb{R}^2,(k'_N,k'_O)\in\mathbb{R}^2_+} d - \phi(-d) + \frac{1}{R}\mathbb{E}\Big[V_{t+1}(s')\Big|z\Big] \tag{7}$$

$$\text{s.t. } b' \leq \theta_t\Big[k'_N + q_t k'_O\Big] \tag{8}$$

$$d = \pi_t(s) - c_F - k'_N - q_t k'_O + b' \tag{9}$$

Use $d_t(s)$, $b'_t(s)$, $k'_{N,t}(s)$, and $k'_{O,t}(s)$ to denote the dividend, borrowing, and capital policies chosen by firm s at date t.

Potential entrants receive their productivity signal z_E and then decide whether to setup.

$$V_t^E(z_E) = \max\{0, V_{S,t}^E(z_E)\} \tag{10}$$

Use $x_{E,t}(z_E) = 0$ to denote that potential entrant with productivity signal z_E chose to setup at date t and $x_{E,t}(z_E) = 1$ that that potential entrant did not. Entrants that setup firms pay setup cost and choose investment, borrowing, and dividend.

$$V^E_{S,t}(z_E) = \max_{(d,b')\in\mathbb{R}^2,(k'_N,k'_O)\in\mathbb{R}^2_+} d - \phi(-d) + \frac{1}{R}\mathbb{E}\Big[V_{t+1}(s')\Big|z_E\Big] \tag{11}$$

$$\text{s.t. } b' \le \theta_t\Big[k'_N + q_t k'_O\Big] \tag{12}$$

$$d = -c_E - k'_N - qk'_O + b' \tag{13}$$

Use $d_{E,t}(z_E)$, $b_{E,t}(z_E)$ $k_{N,E,t}(z_E)$, and $k_{O,E,t}(z_E)$ to denote the dividend, borrowing, and capital policies chosen by entrant with productivity signal z_E.

The firm distribution μ_t over s evolves following operator T^* based on firm-level policies, such that next period's distribution μ_{t+1} is

$$\begin{aligned}\mu_{t+1} &= T^*(\mu_t, x_t(\cdot), k'_{N,t}(\cdot), k'_{O,t}(\cdot), b'_t(\cdot), x_{E,t}(\cdot), k'_{N,E,t}(\cdot), k'_{O,E,t}(\cdot), b'_{E,t}(\cdot))(\hat{z}, \hat{k}_N, \hat{k}_O, \hat{b}) \\ &= T^*_I(\mu_t, x_t(\cdot), k'_{N,t}(\cdot), k'_{O,t}(\cdot), b'_t(\cdot),)(\hat{z}, \hat{k}_N, \hat{k}_O, \hat{b}) \\ &+ T^*_E(x_{E,t}(\cdot), k'_{N,E,t}(\cdot), k'_{O,E,t}(\cdot), b'_{E,t}(\cdot))(\hat{z}, \hat{k}_N, \hat{k}_O, \hat{b})\end{aligned} \tag{14}$$

where operator T^*_I accounts the evolution of firm distribution stemming from incumbents and operator T^*_E accounts the evolution of firm distribution stemming from entrants

$$\begin{aligned}&T^*_I(\mu_t, x_t(\cdot), k'_{N,t}(\cdot), k'_{O,t}(\cdot), b'_t(\cdot),)(\hat{z}, \hat{k}_N, \hat{k}_O, \hat{b}) \\ &\quad= \int [1 - x_t(s)]\mathbb{1}\{\hat{k}_N = k'_{N,t}(s)\}\mathbb{1}\{\hat{k}_O = k'_{O,t}(s)\}\mathbb{1}\{\hat{b} = b'_t(s)\}Q_z(\hat{z}|z_{i,t})\mu_t(ds) \\ &T^*_E(x_{E,t}(\cdot), k'_{N,E,t}(\cdot), k'_{O,E,t}(\cdot), b'_{E,t}(\cdot))(\hat{z}, \hat{k}_N, \hat{k}_O, \hat{b}) \\ &\quad= \int [1 - x_{E,t}(z_E)]\mathbb{1}\{\hat{k}_N = k'_{N,E,t}(z_E)\}\mathbb{1}\{\hat{k}_O = k'_{O,E,t}(z_E)\}\mathbb{1}\{\hat{b} = b'_{E,t}(z_E)\}Q_{E,z}(\hat{z}|z_E)Q_E(dz_E)\end{aligned}$$

Using operators T^*_I and T^*_E, I can define age-specific firm distributions, which I use to calibrate the model in Section 6, as

$$\mu^{(h)}_{t+1} = \begin{cases} T^*_E(x_{E,t}(\cdot), k'_{N,E,t}(\cdot), k'_{O,E,t}(\cdot), b'_{E,t}(\cdot))(\hat{z}, \hat{k}_N, \hat{k}_O, \hat{b}), & \text{if } h = 1 \\ T^*_I(\mu^{(h-1)}_t, x_t(\cdot), k'_{N,t}(\cdot), k'_{O,t}(\cdot), b'_t(\cdot))(\hat{z}, \hat{k}_N, \hat{k}_O, \hat{b}), & \text{if } h \in \{2, 3, ...\} \end{cases}$$

Define aggregate new and old capital before production as

$$K_{N,t} \equiv \int k_N \mu_t(ds) \tag{15}$$

$$K_{O,t} \equiv \int k_O \mu_t(ds) \tag{16}$$

Aggregate old capital next period $t+1$ equals old capital policies of continuing firms and entrant this period t

$$\begin{aligned} K_{O,t+1} &= \int k'_{O,t}(s)[1 - x_t(s)]\mu_t(ds) + \int k'_{O,E,t}(z_E)[1 - x_{E,t}(z_E)]Q_E(dz_E) \\ &= \int \hat{k}_O \mu_{t+1}(d\hat{s}) \end{aligned}$$

For old capital market clearing in period t, the supply of old capital is both aggregate new capital that has depreciated into old capital and deprecated old capital, which equals the old capital demanded by continuing firms and entrants

$$\delta_N K_{N,t} + (1 - \delta_O)K_{O,t} = K_{O,t+1} \tag{17}$$

Define aggregate borrowing, dividends, equity issuance costs, output, investment, fixed costs, and consumption as

$$
\begin{aligned}
B_t &\equiv \int b\mu_t(ds) \\
D_t &\equiv \int [1 - x_t(s)] d_t(s) \mu_t(ds) + \int x_t(s) \pi_t(s) \mu_t(ds) + \int [1 - x_{E,t}(s)] d_{E,t}(s) Q_E(dz_E) \\
\Phi_t &\equiv \int [1 - x_t(s)] \phi(-d_t(s)) \mu_t(ds) + \int x_t(s) \phi(-\pi_t(s)) \mu_t(ds) \\
&\quad + \int [1 - x_{E,t}(s)] \phi(-d_{E,t}(s)) Q_E(dz_E) \\
Y_t &\equiv \int f(z, k_N, k_O; a_t) \mu_t(ds) \\
I_t &\equiv K_{N,t+1} - (1 - \delta_N) K_{N,t} \\
\Upsilon_t &\equiv c_F \int [1 - x_t(s)] \mu_t(ds) + c_E \int [1 - x_{E,t}(z_E)] Q(dz_E) \\
C_t &\equiv Y_t - I_t - \Upsilon_t = D_t - \Phi_t + RB_t - B_{t+1}
\end{aligned}
$$

The first term of aggregate dividends is dividends from continuing firms, the second term is dividends from exiting firms, and the third term is dividends from entrants. The last line states that aggregate consumption can be derived by the resource constraint or the household budget constraint.

Definition: Given an initial firm distribution μ_0, an industry equilibrium is incumbent firm policies $\{x_t(s),\ d_t(s),\ k'_{N,t}(s),\ k'_{O,t}(s),\ b'_t(s)\}$ for all dates t and idiosyncratic states s, potential entrant firm policies $\{x_{E,t}(z_E),\ d_{E,t}(z_E),\ k'_{N,E,t}(z_E),\ k'_{O,E,t}(z_E),\ b'_{E,t}(z_E)\}$ for all dates t and productivity signals z_E, firm distribution $\{\mu_t\}$ for all dates t, and an old capital price sequence $\{q_t\}$ for all dates t such that incumbent firm policies solve (5) and (7), potential entrant policies are the solutions to (10) and (11), the firm distribution follows firm-level policies of (14), and the market for old capital clears (17).

6 Calibration and Computation

First, I externally calibrate standard parameters following the literature. Second, I internally calibrate parameters that govern entry and exit and financial frictions to match steady-state firm dynamics moments where $a_t = 0.0$ and $\theta_t = \bar{\theta}$. Third, I calibrate the aggregate productivity and collateralizability process to target the comovement of aggregate output and borrowing.

Table 2 reports externally calibrated parameters. I calibrate the model so each period is one quarter. Following Khan and Thomas (2013), idiosyncratic productivity is a discretized AR(1) process $z_{i,t} = \rho_z z_{i,t-1} + \varepsilon_{z,i,t}$ where $\varepsilon_z \sim N(0, \sigma_z^2)$.[4] I discretize this process using Tauchen (1986) to seven origin grid points and 15 destination grid points while computing the continuing value function. I then use cubic interpolate value and policy function for the exit choice and firm distribution to 36 grid points. Using both the coarse and fine grids balances computation time and targeting exit rates, which with discrete grids are step functions in the fixed operating cost.

As in Clementi and Palazzo (2016), productivity signals evolve into a firm's initial idiosyncratic productivities following the same AR(1) coefficients as the productivity process itself $z_{i,t} = \mu_z z_{E,i} + \varepsilon_{z,i,t}$ where $\varepsilon_{z,i,t} \sim N(0, \sigma_z^2)$. I assume that entrant productivity signals are distributed truncated Pareto with tail parameter ξ over support $[-\bar{z}_E, \bar{z}_E]$.

$$Q_E(z_E) \equiv \frac{\tilde{Q}_E(z_E)}{\sum_{\hat{z}_E \in Z_E} \tilde{Q}_E(\hat{z}_E)} \text{ where } \tilde{Q}_E(z_E) \equiv e^{(-\xi-1)z_E}$$

I use 50 equally-spaced grid points for entrant productivity signals. I choose $\bar{z}_E = 1.11$, so that the entrant with productivity signal $z_E = \bar{z}_E$ ($z_E = -\bar{z}_E$) has a 99% probability of drawing the highest (lowest) z grid point.

Instead of tracking k_N, k_O, b separately, I track the firm distribution on using net worth after production and repayment $\pi(s)$ defined in equation 6. Similar to the using both fine and coarse grids of idiosyncratic productivity grid, I solve the continuing firm on a relatively net worth coarse grid with 31 log-spaced points between 0 and $\exp(9) \approx 8000$ and solve the exit decision and the firm distribution using a finer grid of 301 points over the same interval.

[4] I adjust them annual estimates from Khan and Thomas (2013) to quarterly frequency.

Table 2: External Parameters

Description	**Source**	**Parameter**	**Value**
Discount Rate	Annual interest rate = 4%	R	1.01
Span of Control	Standard Parameter	α	0.6
Elasticity of Substitution Between Capital	Lanteri (2018)	ϵ	5.0
New Capital Weight	Lanteri (2018)	ν	0.5
New Capital Depreciation	Total annual depreciation rate = 10%	δ_N	0.055
Old Capital Depreciation		δ_O	0.055
Idiosyncratic Productivity Persistence	Khan and Thomas (2013)	ρ_z	0.9
Idiosyncratic Productivity Volatility	Khan and Thomas (2013)	σ_z	0.068
Collateralizability	Li, Whited, and Wu (2016)	$\bar{\theta}$	0.5

Second, I internally calibrate the parameters including the Pareto tail of entrant productivity signals ξ, the entry cost c_E, the fixed operating c_F, and the scale of equity issuance costs ϕ_0 reported in the top half of Table 3 using simulated method of moments to match cross-sectional moments reported in Table 4. I match the steady-state size of entrants and exiting firms relative to incumbent firms (in terms of output) from Lee and Mukoyama (2015). I also match establishment exit hazard rate by establishment age from the Census Business Dynamics Statistics between 2000 and 2020. I also match average equity issuance over capital from nonfinancial public firms using Compustat data from 2000-2020 as in Hennessy and Whited (2007).

Table 3: Internal Parameters

Description	**Parameter**	**Value**
Entrant Productivity Signal Distribution	ξ	0.47
Entry Cost	c_E	10.02
Fixed Operating Cost	c_F	32.43
Equity Issuance Cost	ϕ_0	0.0006
Aggregate Productivity Persistence	ρ_a	0.71875
Aggregate Productivity Volatility	σ_a	0.005
Credit Shock Persistence	ρ_ω	0.71875
Credit Shock Volatility	σ_ω	0.008
Correlation of Aggregate Productivity and Credit Shock	$\rho_{a\omega}$	-0.797

Table 4: Targeted Steady-State Moments

Moment	**Source**	**Data (%)**	**Model (%)**
Mean Entrant Size	Lee & Mukoyama (2015)	60	54.53
Mean Exiting Firm Size	Lee & Mukoyama (2015)	49	44.13
Exit Rate (Ages 1–5)	Census BDS	16.06	13.15
Exit Rate (Ages 6–10)	Census BDS	9.79	3.76
Exit Rate (Ages 11–30)	Census BDS	6.95	4.79
Mean Equity Issuance	Compustat	0.3780	0.5869

I calibrate the steady-state by drawing 4-dimensional Sobel sequence with 500 draws and solving the stationary equilibrium in the hypercube of $\xi \in [0, 0.5]$, $c_F \in [28, 38]$, $c_E \in [5, 12]$, and $\phi_0 \in [0.0, 0.03]$, then choose the best fit to the steady-state moments. Formally, I estimate model parameters $\Theta_1 \equiv (\xi, c_E, c_F, \phi_0)$ by minimizing the distance between model-implied moments and empirical moments using a generalized method of moment (GMM) objective function. Let the vector of empirical moments be

$$\mu_1^{\text{data}} = \begin{bmatrix} \text{Mean Entrant Size} \\ \text{Mean Exiting Firm Size} \\ \text{Exit rate (Age 1–5)} \\ \text{Exit rate (Age 6–10)} \\ \text{Exit rate (Age 11–30)} \\ \text{Mean Equity Issuance} \end{bmatrix}$$

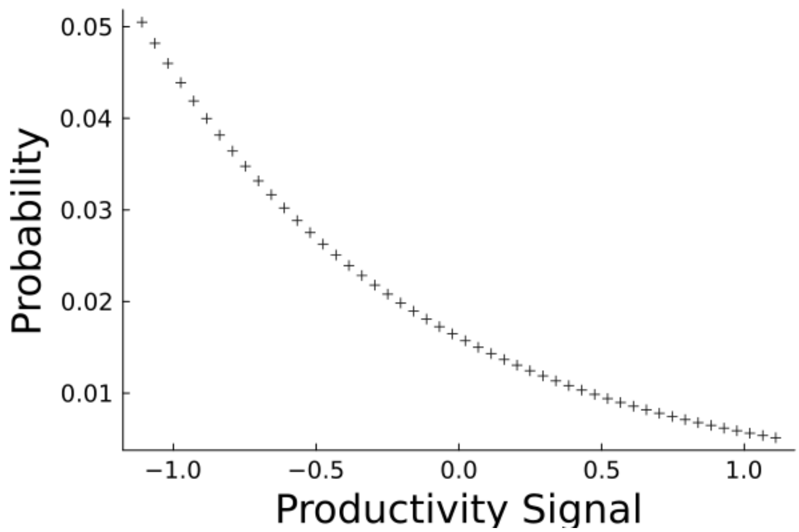
$$\mu_t^{\text{model}}(\Theta_1) = \begin{bmatrix} \frac{\int\int f(z, k_N'(z_E), k_O'(z_E); 0) Q_{E,z}(z|z_E) Q_E(z_E)}{\int f(z, k_N, k_O; 0)\mu(ds)} \\ \frac{\int x(s) f(z, k_N, k_O; 0)\mu(ds)}{\int f(z, k_N, k_O; 0)\mu(ds)} \\ \frac{\sum_{h\in[1,5]} \int x(s)\mu^{(h)}(ds)}{\sum_{h\in(1,5)} \int \mu^{(h)}(ds)} \\ \frac{\sum_{h\in[6,10]} \int x(s)\mu^{(h)}(ds)}{\sum_{h\in(6,10)} \int \mu^{(h)}(ds)} \\ \frac{\sum_{h\in[11,30]} \int x(s)\mu^{(h)}(ds)}{\sum_{h\in(11,30)} \int \mu^{(h)}(ds)} \\ \frac{-\int d(s)\mathbb{1}\{d(s)<0\}\mu(ds)}{\int (k_N + k_O)\mu(ds)} \end{bmatrix}$$

I use the moment deviation as the element-wise relative deviation to emphasize the relative fit across all moments

$$m_1(\Theta_1) = \frac{\mu_1^{\text{model}}(\Theta_1) - \mu^{\text{data}}}{\mu_1^{\text{data}}}$$

and internally calibrated parameters reported in Table 3 minimize the GMM objective function $\Theta_1^* = \arg\min m(\Theta_1)^\top W m(\Theta_1)$ where $W = I$. Table 4 shows a decent fit. Entrants and exiters are roughly half the size of incumbents and exit rates are about double for young firms relative to older firms Figure 3 shows entrant signal distribution $Q_E(z_E)$ for the calibrated Pareto tail parameter ξ.

Figure 3: Entrant Productivity Signal Distribution



In the third stage of calibration, I internally calibrate productivity and credit shocks, reported in the bottom half of Table 3, as correlated AR(1) processes

$$a_t = \rho_a a_{t-1} + \varepsilon_{a,t} \tag{18}$$

$$\omega_t = \rho_\omega \omega_{t-1} + \varepsilon_{\omega,t} \tag{19}$$

$$\begin{pmatrix} \varepsilon_{a,t} \\ \varepsilon_{\omega,t} \end{pmatrix} \sim N\left(\begin{pmatrix} 0 \\ 0 \end{pmatrix}, \begin{pmatrix} \sigma_a^2 & \rho_{a\omega}\sigma_a\sigma_\omega \\ \rho_{a\omega}\sigma_a\sigma_\omega & \sigma_\omega^2 \end{pmatrix} \right) \tag{20}$$

To compute business cycle dynamics, I use the following sequence-space approach from Boppart, Krusell, and Mitman (2018). For each variable of interest X, I compute steady-state aggregate X_{SS} with $a_{SS} = 0$ and $\theta_{SS} = \bar{\theta}$. I then compute two transitions $\{X_t^a\}_{t=0}^T$ and $\{X_t^\omega\}_{t=0}^T$ in response to unanticipated shocks $\{a_t\}_{t=0}^T$ and $\{\omega_t\}_{t=0}^T$, respectively, where $a_t = \rho_a^t a_0$ and $\omega_t = \rho_\omega^t \omega_0$ with $a_0 = -0.01$ and $\omega_0 = -0.2$ until $T = 30$. At T, I assume the transition returns to the steady state. To compute each transition, I guess that the path of old capital prices $\{q_t^{(1)}\}$ is the steady-state price q_{SS} and solve the firm problem backward. Then I move forward on the transition. In each period t, I solve for the old capital price $\hat{q}_t^{(1)}$ that satisfies market-clearing when firms expect the future path of old capital prices to follow $\{q_t^{(1)}\}$. For the next iteration, the guessed path of old capital prices for solving backward are thus implied by market-clearing $q_t^{(2)} = \hat{q}_t^{(1)}$. I repeat this process n times where $\{q_t^{(n)}\}$ and $\{\hat{q}_t^{(n)}\}$ converge in sup norm. With the paths of X_t^a and X_t^ω along each transition, I calculate impulse response function $\{h_t^{X,a}, h_t^{X,\omega}\}_{t=0}^T$ as

$$h_t^{a,X} = \frac{\log(X_t^a) - \log(X_{SS})}{a_0} \tag{21}$$

$$h_t^{\omega,X} = \frac{\log(X_t^\omega) - \log(X_{SS})}{\omega_0} \tag{22}$$

Finally, I simulate a path of random shock innovations $\{\tilde{a}_t, \tilde{\omega}_t\}_{t=1}^S$ following exogenous dynamics in (18), (19), and (20). The path of the aggregate variable $\{\tilde{X}_t\}_{t=1}^S$ is the moving average of shocks with the impulse response as weights

$$\log(\tilde{X}_s) = \log(X_{SS}) + \sum_{t=0}^{T} h_t^{a,X} \tilde{a}_{s-t} + \sum_{t=0}^{T} h_t^{\omega,X} \tilde{\omega}_{s-t}$$

I compute the impulse response functions (21) and (22) for a 1-dimensional Sobel sequence with 10 draws each of $\rho_a \in [0.0, 0.5]$ and $\rho_\omega \in [0.0, 0.5]$ and then, for each (ρ_a, ρ_ω), simulate aggregate output and borrowing for a 3-dimensional Sobel sequence with 1000 draws for $\sigma_a \in [0, 0.01]$, $\sigma_\omega \in [0, 0.01]$, and $\rho_{a\omega} \in [-1, 0]$.

I match the volatility, persistence, and correlation of the cycle of log real output and log real aggregate borrowing using a Hodrick-Prescott filter with smoothing parameter of 1,600. To measure output, I use the quarterly average of industrial production from the Federal Reserve Board G.17 Industrial Production and Capacity Utilization release deflated by GDP. For robustness, I also look at real GDP in place of industrial production. Real GDP is less volatile than industrial production at 1.38% and less persistent with autocorrelation of 69.83%; omitting 2020:Q2, volatility drops to 1.22% and persistence increases to 81.06%. To measure real aggregate borrowing, I use the noncorporate nonfinancial debt securities and loans from Table D.3 of the Flow of Funds deflated by GDP. For robustness, I look at both total nonfinanical debt and corporate debt with and without 2020:Q2. Volatility ranges from 2.30% to 2.75% and persistence ranges from 84.20% to 92.62%. Results are unlike to meaningful change matching these other moments. Contemporaneous correlation between the cycle of industrial production and the cycle of noncorporate nonfinancial borrowing is negative at -26.68%, which the model struggles to match, so I match the correlation between output and lagged borrowing by a year.

Table 5: Targeted Time-Series Moments

Moment	**Data (%)**	**Model (%)**
Output Volatility	2.76	1.85
Output Persistence	81.06	90.85
Borrowing Volatility	2.40	2.86
Borrowing Persistence	82.36	85.99
Correlation of Output & Lagged Borrowing	41.20	15.90

As in Clementi and Palazzo (2016), table 3 and table 5 shows that this model generates persistence with the aggregate productivity persistence being much less than aggregate output persistence.

7 Evaluation

Figure 4 shows policy and value functions for incumbent firms. The capital (total and share of capital that is old is plotted), borrowing, and dividend policy functions resemble Lanteri and Rampini (2023). More financially constrained firm using relatively more old capital. Relative to their exogenous exit setup is the exit policy function where firms in the black region exit and the firms in the white region continue. Low productivity firms exit and the exit threshold is increasing as net worth is lower. The final panel shows value functions. It depicts the continuing value function $V^C(z, w)$ in solid lines and the exiting value function $V^X(z, w) = w$ in dashed lines. For a given productivity, a firm with a net worth below the dashed line exits and continues if it is above. The first stage value function $V(z, w)$ is the upper envelope of $V^C(z, w)$ and $V^X(z, w)$. As productivity gets higher, $V^C(z, w)$ and $V^X(z, w)$ are equal at higher net worth levels, creating the exit policy figure on the left.

Figure 4: Incumbent Policy and Value Functions

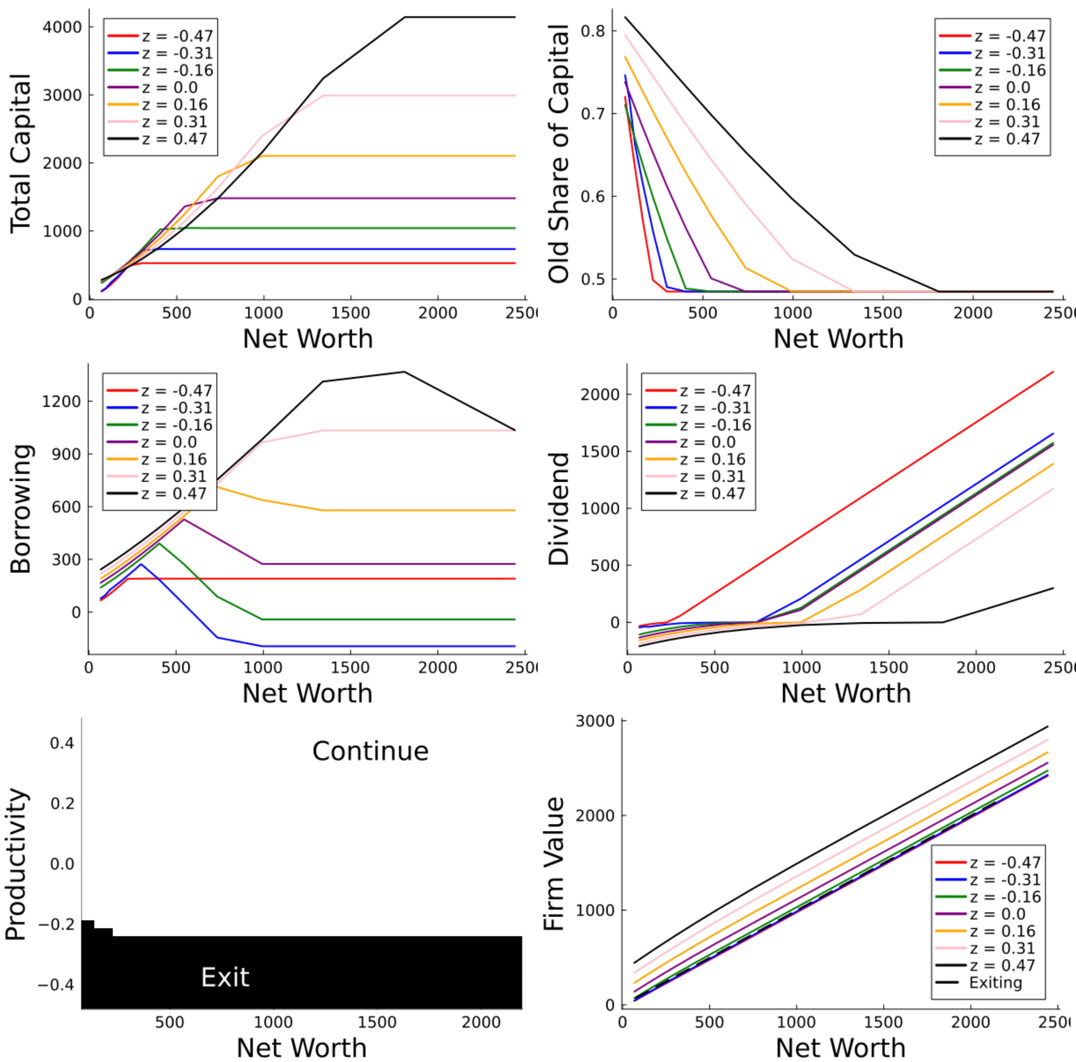


Figure 5 shows policy and value functions for entrants. Entrants who have no net worth, rely heavily on old capital. For entrants, total capital and borrowing is monotonically increasing in the productivity signal, while the dividend is monotonically decreasing. An entrant with a high productivity signal wants to take advantage of the high expected productivity next period, so issues more equity to fund investment. Entrants with low productivity signals choose not to set and entrants with high productivity signals choose to set up.

Figure 5: Entrant Policy and Value Functions

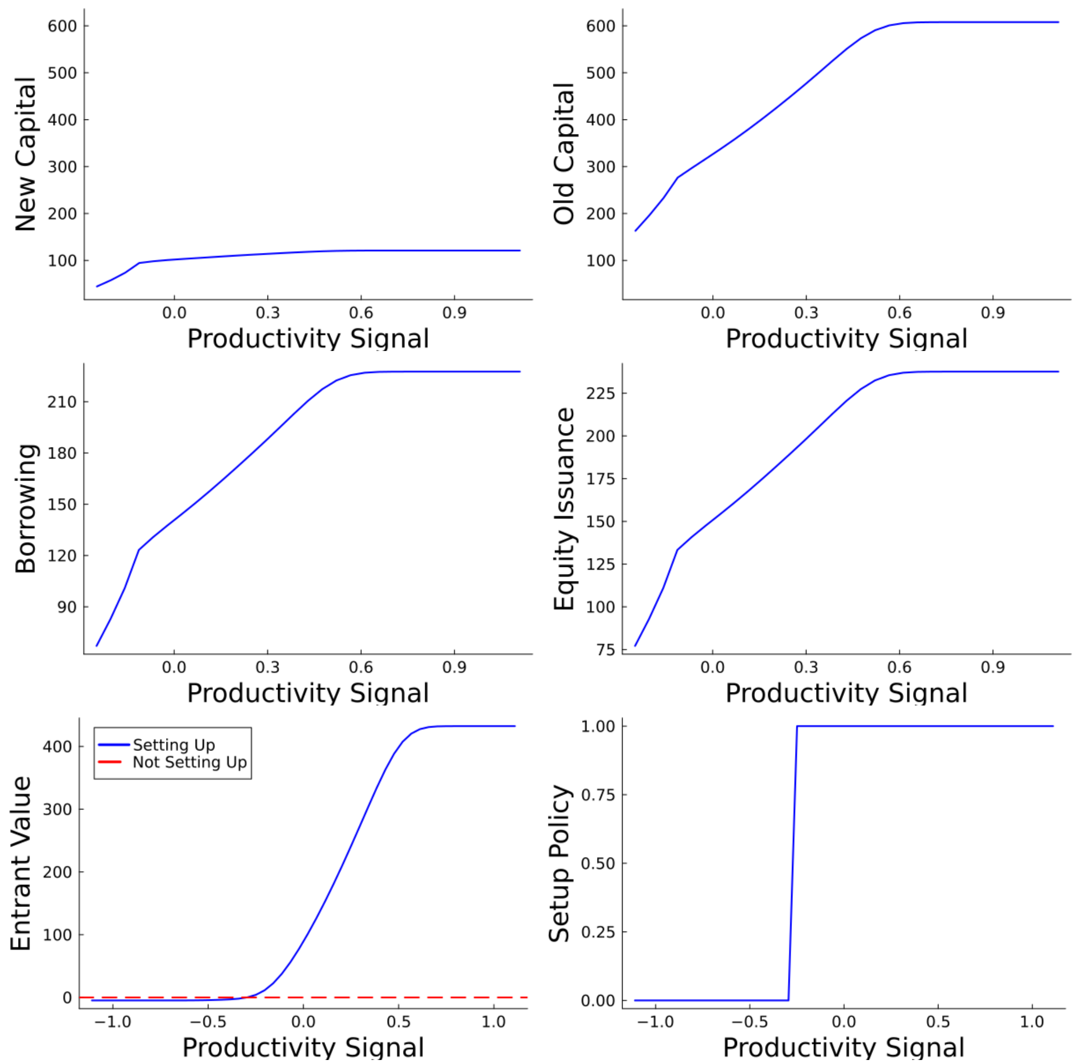


Figure 6 shows impulse response functions for old capital prices as well as aggregates in response to productivity shocks (blue solid lines) and credit shocks (red dashed lines). Old capital prices are procyclical. Generally, the effect of credit shocks is smaller. These impulse response functions are interpreted as the percent change in a particular variable to a one-percent shock to either productivity or the collateralizability of capital.

Figure 6: Impulse Response Functions

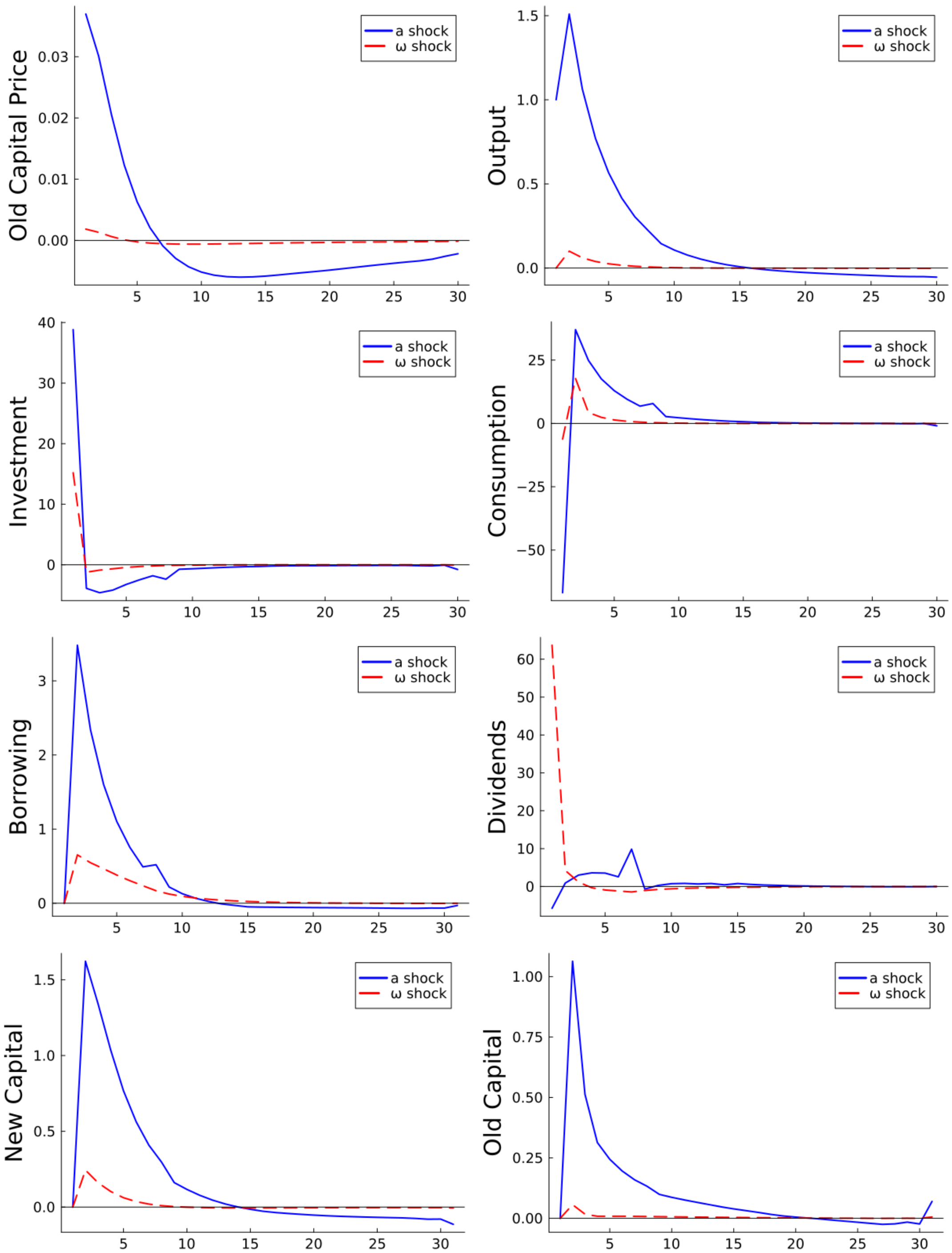


Table 6 shows the volatility, output correlation, and autocorrelation of secondary market capital prices from the model relative to the used vehicle price index over the consumption price index as defined in section 3. I simulate a long path of the old capital price, log it, and apply an Hodrick-Prescott filter with smoothing lambda of 1,600. The model delivers a fraction of the volatility of secondary market prices observed in the data, but the model does deliver procyclical secondary market prices with reasonable persistence.

Table 6: Secondary Capital Market Prices

%	**Model**	**Data** $(\frac{Used}{CPI})$	**Data** $(\frac{Used}{New})$
Volatility	0.0527	4.84	4.46
Output Correlation	94.76	23.08	23.29
Autocorrelation	89.55	88.00	86.36

In table 7, I compare the relative volatility, correlation with aggregate output and autocorrelation of aggregate consumption measured by real personal consumption expenditures and aggregate investment measured by real gross private domestic investment. I use real GDP for output and for investment, and for consumption. Overall, the model produces consumption and investment that is much more volatile and less correlated with output than the data. The high volatility is a result of the initial spikes seen in the respective impulse response functions.

Table 7: Consumption and Investment Business Cycle Moments

%		**Consumption** C_t	**Investment** I_t
Volatility	Model	44.96	16.59
	Data	2.30	5.86
Output Correlation	Model	16.40	15.21
	Data	63.14	87.48
Autocorrelation	Model	55.03	53.87
	Data	22.51	80.55

Table 8 displays business cycle dynamics of the exit and entry rates as well as mean exiting firm size relative to the mean incumbent firm size and mean entrant relative to the mean incumbent firm size. The model delivers countercyclical exit, but also slightly countercyclical entry, when in the data it is procyclical (Lee and Mukoyama 2015). Breaking the entry rate down into its numerator and denominator, the mass of entrants, the numerator, is acyclical while the total mass of firms is procyclical because exit is strongly countercyclical. Thus, the entry rate ends up being countercyclical. The size of exiting firms is procyclical, which matches the data. Again, the size of entrant firm is counterfactual being countercyclical.

Table 8: Entry and Exit Across Business Cycle

%		Exit Rate	Entry Rate	Mean Exiting Firm Size	Mean Entrant Firm Si
Volatility	Model	0.53	0.11	0.95	1.08
	Data				
Output Correlation	Model	-89.12	-22.61	47.14	-35.10
	Data	−	+	+	−
Autocorrelation	Model	82.57	93.92	71.60	71.93
	Data				

8 Results

8.1 Financial Amplification of Secondary Market Capital Prices

In this section, I decompose secondary market capital price volatility into fundamental volatility and amplification from financial frictions. To do so, I compare the volatility of secondary-market capital price in the baseline model with the volatility of prices in a frictionless benchmark. In the frictionless benchmark, there are no equity issuance costs $\phi_0 = 0$ and, since the Modigliani Miller theorem holds, there is no borrowing $\theta = 0$. Table 9 shows that the volatility of secondary market capital prices in the frictionless benchmark accounts 90 percent of the volatility of secondary market capital prices in baseline model, so the remaining 10 percent can be attributed to amplification from financial frictions. This shows that financial frictions plays a role in amplifying secondary market capital price volatility.

Table 9: Secondary Market Capital Price Volatility Decomposition

Description	Variable	Frictionless Benchmark	Baseline
Equity Issuance Cost	ϕ_0	0	0.1
Collateralizability	θ	0	0.5
Secondary Market Capital Price Volatility	$\sigma(q_t) \times 10^4$	2.018	2.231
Contribution		90.464	9.536

8.2 Output Amplification from Secondary Market Capital Price Volatility

In this section, I decompose the volatility of aggregate output to quantify the role of secondary market capital price volatility. To do so, I compute two alternative models. In the first alternative model, I hold the secondary market price of capital at its steady-state level. In the second alternative model, I set the secondary market price to evolve in response to aggregate shocks following impulse responses from the frictionless benchmark:

$$\log(\tilde{q}_t^{frictionless}) \equiv \log(q_{SS}^{baseline}) + \sum_{t=0}^{T} h_t^{q,frictionless} \tilde{a}_{s-t}$$

Note that for both of these alternative models, market-clearing in old capital market is violated unlike in the baseline and frictionless benchmark. I find that when the secondary market capital price is held at its steady-state value, the volatility of aggregate output accounts is 111 percent of the baseline. Allowing the secondary market capital price to evolve based on frictionless dynamics attenuates 8 percent of aggregate output volatility of the baseline. The remaining 3 percent can be attributed to the role of the amplified secondary market capital price volatility stemming from financial frictions in attenuating shocks. This suggests that both fundamental volatility and the financial amplification of secondary market capital prices play a role in attenuating the volatility of aggregate output in response to productivity shocks.

Table 10: Aggregate Output Volatility Decomposition

Description	Variable	Steady-State	Frictionless Dynamics	Baseline
Secondary Market Capital Price	q_t	$q_{SS}^{baseline}$	$\tilde{q}_t^{frictionless}$	$q_t^{baseline}$
Aggregate Output Volatility	$\sigma(Y_t)$	3.298	3.065	2.967
Contribution		111.158	-7.864	-3.294

9 Conclusion

This paper presents a quantitative theory of investment and capital reallocation across heterogeneous firms with financial frictions, endogenous entry and exit, and aggregate shocks. I match this model to important cross-sectional and timeseries moments for the dynamics of the universe of U.S. firm. The model produces reasonable untargetted business cycle moments. I use the model to decompose the volatility of secondary market capital prices into fundamental volatility, that is the volatility of this price in a model without financial frictions, and the remaining financial amplification. I find that financial amplification is sizable. Furthermore, I look at the role of secondary market capital price volatility has in driving aggregate output volatility. Again, I find that its role is meaningful.

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A Uniform Commercial Code Filings

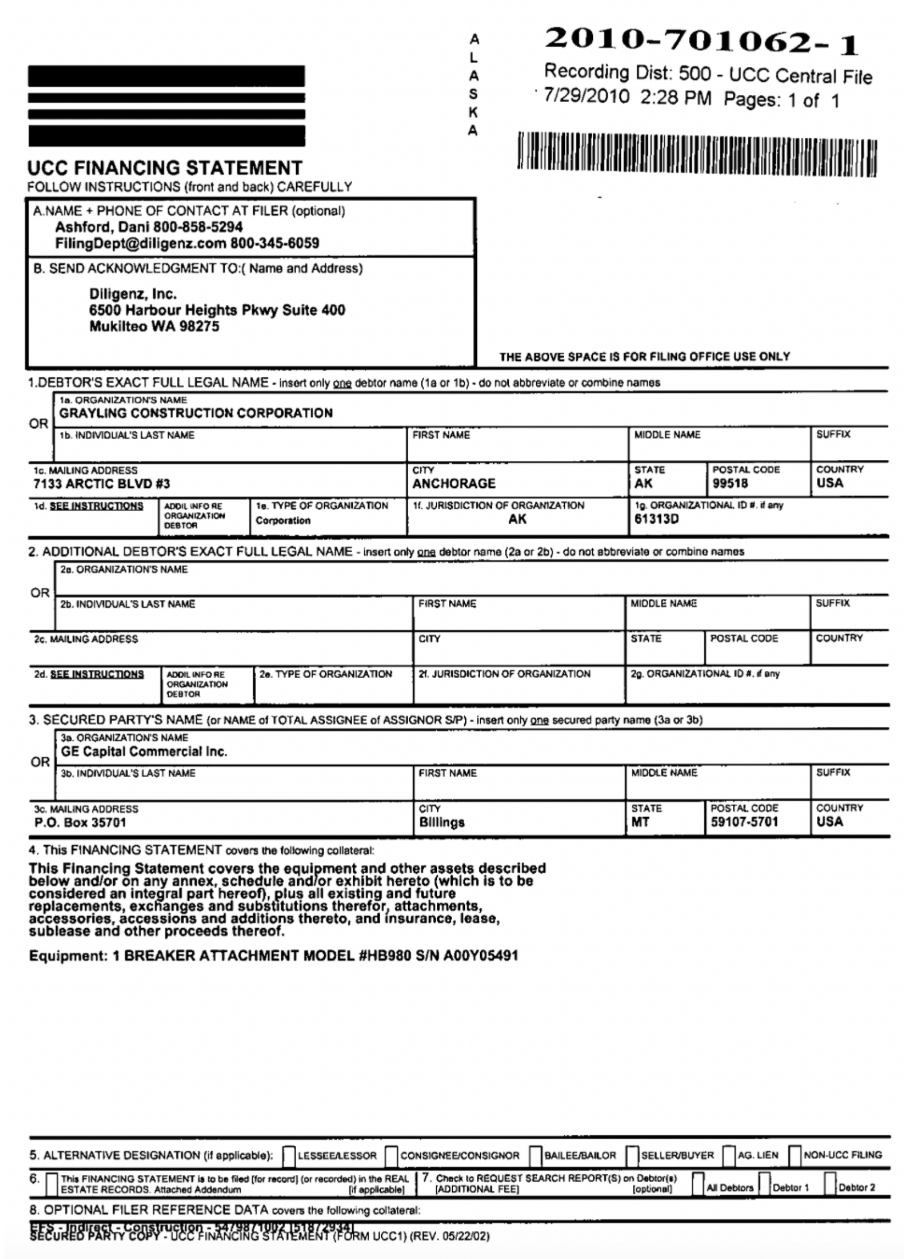
To construct this data, I purchase bulk data downloads of Uniform Commercial Code (UCC) filings from five U.S. states: California, Colorado, Connecticut, Florida, and Virginia. When borrowing is collateralized by a debtor’s assets, lenders can file UCC financing statements with the Secretaries of State in the state that the debtor is incorporated. This filing serves two purposes. First, it perfects the lender’s security interest in the event of bankruptcy or default. And second, it publicly indicates the lender’s security interest in a debtor’s assets to other lenders.

UCC filings cover collateral in the form of tangible property like inventories, equipment, fixtures, intangible property like accounts receivable, chattel paper, intellectual property, patents, trademarks, copyrights, and goodwill, investment property like stocks, bonds, and securities, and deposit accounts. Real estate, personal titled vehicles, aircraft (covered by the Federal Aviation Administration), and boats and ships over 26 feet (covered by the U.S. Coast Guard), and some railroad cars (if federally regulated under the Surface Transportation Board) are examples of assets not covered by UCC financing statements. UCC filings themselves do not prevent the sale of the collateral, but such covenants are likely in the underlying agreements.

The UCC financing statements give a lender priority over the debtor’s specified assets for a period of five year. Within the five-year window, the lender can file a termination statement, which ends the lender's priority to the debtor's assets, or a continuation statement, which extends the lender's priority for an additional five years. Fees associated with UCC filings are small; in my sample of states, range from \$8 in Colorado to \$50 in Connecticut. There are backend integration system for lender's IT systems.

An example UCC financing statement is shown in Figure A1, which was downloaded from the State of Alaska Department of Natural Resources Record's Office. This statement was filed on behalf of GE Capital Commercial Inc., which is the captive financing arm of equipment manufacturer General Electric. It indicates a security interest in a breaker attachment, which can go on the arm of an excavator in order to break concrete. The breaker attachment is owned by Grayling Construction Corporation, which is a small residential construction company in Anchorage, Alaska.

Figure A1: Example Uniform Commercial Code Financing Statement



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2010-701062-1
Recording Dist: 500 - UCC Central File
7/29/2010 2:28 PM Pages: 1 of 1

UCC FINANCING STATEMENT
FOLLOW INSTRUCTIONS (front and back) CAREFULLY

A. NAME + PHONE OF CONTACT AT FILER (optional)
Ashford, Dani 800-858-5294
FilingDept@diligenz.com 800-345-6059

B. SEND ACKNOWLEDGMENT TO:(Name and Address)
Diligenz, Inc.
6500 Harbour Heights Pkwy Suite 400
Mukilteo WA 98275

THE ABOVE SPACE IS FOR FILING OFFICE USE ONLY

1. DEBTOR'S EXACT FULL LEGAL NAME - insert only one debtor name (1a or 1b) - do not abbreviate or combine names

1a. ORGANIZATION'S NAME: GRAYLING CONSTRUCTION CORPORATION
OR
1b. INDIVIDUAL'S LAST NAME | FIRST NAME | MIDDLE NAME | SUFFIX

1c. MAILING ADDRESS	CITY	STATE	POSTAL CODE	COUNTRY
7133 ARCTIC BLVD #3	ANCHORAGE	AK	99518	USA

1d. SEE INSTRUCTIONS	ADD'L INFO RE ORGANIZATION DEBTOR	1e. TYPE OF ORGANIZATION	1f. JURISDICTION OF ORGANIZATION	1g. ORGANIZATIONAL ID #, if any
		Corporation	AK	61313D

2. ADDITIONAL DEBTOR'S EXACT FULL LEGAL NAME - insert only one debtor name (2a or 2b) - do not abbreviate or combine names

2a. ORGANIZATION'S NAME
OR
2b. INDIVIDUAL'S LAST NAME | FIRST NAME | MIDDLE NAME | SUFFIX

2c. MAILING ADDRESS | CITY | STATE | POSTAL CODE | COUNTRY

2d. SEE INSTRUCTIONS | ADD'L INFO RE ORGANIZATION DEBTOR | 2e. TYPE OF ORGANIZATION | 2f. JURISDICTION OF ORGANIZATION | 2g. ORGANIZATIONAL ID #, if any

3. SECURED PARTY'S NAME (or NAME of TOTAL ASSIGNEE of ASSIGNOR S/P) - insert only one secured party name (3a or 3b)

3a. ORGANIZATION'S NAME: GE Capital Commercial Inc.
OR
3b. INDIVIDUAL'S LAST NAME | FIRST NAME | MIDDLE NAME | SUFFIX

3c. MAILING ADDRESS	CITY	STATE	POSTAL CODE	COUNTRY
P.O. Box 35701	Billings	MT	59107-5701	USA

4. This FINANCING STATEMENT covers the following collateral:

This Financing Statement covers the equipment and other assets described below and/or on any annex, schedule and/or exhibit hereto (which is to be considered an integral part hereof), plus all existing and future replacements, exchanges and substitutions therefor, attachments, accessories, accessions and additions thereto, and insurance, lease, sublease and other proceeds thereof.

Equipment: 1 BREAKER ATTACHMENT MODEL #HB980 S/N A00Y05491

5. ALTERNATIVE DESIGNATION (if applicable): LESSEE/LESSOR CONSIGNEE/CONSIGNOR BAILEE/BAILOR SELLER/BUYER AG. LIEN NON-UCC FILING

6. This FINANCING STATEMENT is to be filed [for record] (or recorded) in the REAL ESTATE RECORDS. Attached Addendum [if applicable]

7. Check to REQUEST SEARCH REPORT(S) on Debtor(s) [ADDITIONAL FEE] [optional] All Debtors Debtor 1 Debtor 2

8. OPTIONAL FILER REFERENCE DATA covers the following collateral:

EFS - Indirect - Construction - 54798710002 [518/2934]

SECURED PARTY COPY - UCC FINANCING STATEMENT (FORM UCC1) (REV. 05/22/02)

The bulk downloads include name and address of the debtor, name and address of lender, and whether the filing is initial or amendment. I collapse amendments to get the initial filing date and the lapse date, which is either five years after the initial filing or the final continuation filing or the date of a termination filing.[5] On average, filings are active for on

[5]Other types of amendments like filings to change debtors, lenders, or collateral exist, but constitute a minuscule fraction of the filings, so I ignore them.

average 5.89 years and the bottom 75th percent is 5 years, 95th percentile is 10 years, and next 99th percentile is 15 years. With I clean lender names and categorize lenders as banks, captive financing arm of an equipment manufacturer, credit union, agricultural financing collectives, or other with numbers of filings by each lender type reported in Table A1.[6] If multiple lenders are listed, I prioritize large U.S. banks, then banks, then other categorized lender, then other. Then, I merge the UCC filings with state-level business registry for two purposes: (1) to filter out non-business organizations like churches, universities, and charities, and (2) to get the data of incorporation. If available, I merge on federal employer identification number (FEIN), then use fuzzy-name matching for the remaining filings. For filings with multiple debtors, I use the incorporate date of the oldest debtor that is matched to the business registry. Bulk downloads, with the exception of Colorado, do not include information about the collateral, so I scrape the PDFs of the filings from the website of the respective Secretary of State and use the Tesseract Open Source Optical Character Recognition Engine to digitize them.

Table A1: Count of Filings by Lender Type

Lender Type	Number of Filings
Bank	27,004
Captive	8,143
Credit Union	170
Lease	3,779
Production Credit Association	299
Other	19,329

For each filings, the information about the underlying collateral is an unstructured string. To parse information about the collateral, I search each collateral description for 17-digit alphanumeric serial numbers. A variety of equipment manufacturers, largely vehicle manufacturers, construction equipment manufacturers, and agricultural equipment manufacturers, use this standardized structure of their serial numbers.[7] To read a 17-digit serial number, digits 1 through 3 indicate the World Manufacturer Identifier or similar, digits 4 through 8

[6] I filter out filings that are IRS or state tax liens or list other federal, state, or municipal government as the secured parties.

[7] For example, this convention is the same as vehicle identification numbers (VIN).

indicate the model, digit 9 indicate a check digit, digit 10 represent the model year, digit 11 indicate the manufacturing plant, and digits 12 through 17 are unique to each piece of equipment. Importantly, digit 9, the check digit, that insures that the parsed 17-digit alphanumeric word truly corresponds to a serial number.[8] For example, the serial number for a 2005 Vermeer 1000XL Brush Chipper is

$$\underbrace{\texttt{1vr}}_{\text{World Manufacturer Identifier}}\ \overbrace{\texttt{y1119}}^{\text{model description}}\ \underbrace{\texttt{0}}_{\text{check digit}}\ \overbrace{\texttt{5}}^{\text{model year}}\ \underbrace{\texttt{1}}_{\text{plant code}}\ \overbrace{\texttt{006269}}^{\text{sequential product number}}$$

I then merge with World Manufacturer Identifiers from SAE International and hand-collected other manufacturer identifiers. To something like an industry fixed effect, I then hand-categorize manufacturers into agricultural (e.g., John Deere), construction (e.g., Vermeer), transportation/logistics (e.g., Peterbilt), and other. A row in the resulting data set is a machine that is collateral for a firm at a point in time. Table A2 provides an overview of the dataset.

Table A2: Dataset Overview

Metric	Count
Total Observations	58,724
Unique Filings	33,539
Unique Firms	19,423
Unique Lenders	2,046
Unique Manufacturers	315
Unique Machines	56,035
Median Observations per Firm	1

Other papers, like Ma, Murfin, Pratt (2022) and Darmouni and Sutherland (2024), use commercially available version of UCC filings. Gopal (2023) and Lambert and Schindler (2024) also collect UCC filings directly from states.

A general concern about selection pervades interpreting UCC filings as a view into firms' investment because two scenarios can show up in UCC filings. First, purchase-money security interests to finance purchasing equipment and, second, working capital loans where

[8] The other 16 digits of the serial number are assigned a numeric value based on each letter/number (e.g., "E" are assigned a value of 5) and weighted (e.g., the numeric value in the third digits gets weight of 6), then summed together then divided by 11, and the remainder equals the ninth digit.

already-owned equipment is being putting up as collateral. It is not possible to distinguish between these two scenario. However, legal guidance about the specificity of collateral description is reassuring. If purchase-money security interest tied to specific assets show up in UCC filings, then specific collateral is listed in the description. So we're worried about working capital loans also listing specific equipment. In general, working capital loans are secured by a blanket lien of the borrowers' assets, so are typified by general collateral descriptions, which do not list specific assets. Fortunately, the process of identifying the serial numbers is reassuring. Indeed, after *Ring v. First Niagara Bank* when an overly collateral description prevent a lender from acquiring the borrower's asset in default, "less is more" recommendations are made when describing collateral.[9]

B Illustrative Model Characterization

Marginal products of new and old capital are as follows

$$f_1(k_N, k_O; a) = \frac{\alpha f(k_N, k_O; a)}{g(k_N, k_O)} \left(\frac{\nu g(k_N, k_O)}{k_N} \right)^{\frac{1}{\epsilon}}$$

$$f_2(k_N, k_O; a) = \frac{\alpha f(k_N, k_O; a)}{g(k_N, k_O)} \left(\frac{(1-\nu) g(k_N, k_O)}{k_O} \right)^{\frac{1}{\epsilon}}$$

[9] For example, `https://www.lowenstein.com/media/3424/bc-apr17-nathan.pdf`.

Other aggregate quantities are defined as

$$\begin{aligned}
Y_t &\equiv \mu_{t-1} f(k^{E,1}_{N,t}, k^{E,1}_{O,t}; a_t) + \mu_{t-2} f(k^{E,2}_{N,t}, k^{E,2}_{O,t}; a_t) \\
&+ \upsilon_{t-1} f(k^{I,1}_{N,t}, k^{I,1}_{O,t}; a_t) + \upsilon_{t-2} f(k^{I,2}_{N,t}, k^{I,2}_{O,t}; a_t) \\
I_t &\equiv \mu_t k^{E,1}_{N,t+1} + \mu_{t-1}[k^{E,2}_{N,t+1} - (1-\delta)k^{E,1}_{N,t}] - \mu_{t-2}(1-\delta)k^{E,2}_{N,t} \\
&+ \upsilon_t k^{I,1}_{N,t+1} + \upsilon_{t-1}[k^{I,2}_{I,t+1} - (1-\delta)k^{I,1}_{N,t}] - \upsilon_{t-2}(1-\delta)k^{I,2}_{N,t} \\
D_t &\equiv \mu_{t-2} d^{E,2}_t + \mu_{t-1} d^{E,1}_t + \mu_t d^{E,0}_t \\
&+ \upsilon_{t-2} d^{I,2}_t + \upsilon_{t-1} d^{I,1}_t + \upsilon_t d^{I,0}_t \\
B_t &\equiv \mu_{t-1} b^{E,1}_t + \mu_{t-2} b^{E,2}_t + \upsilon_{t-1} b^{I,1}_t + \upsilon_{t-2} b^{I,2}_t \\
\Phi_t &= \mu_{t-2}\phi(-d^{E,2}_t) + \mu_{t-1}\phi(-d^{E,1}_t) + \mu_t\phi(-d^{E,0}_t) \\
&+ \upsilon_{t-2}\phi(-d^{I,2}_t) + \upsilon_{t-1}\phi(-d^{I,1}_t) + \upsilon_t\phi(-d^{I,0}_t) \\
W_t &\equiv \upsilon_t w^I \\
C_t &\equiv D_t - \Phi_t - W_t + RB_t - B_{t+1}
\end{aligned}$$

Solve the firm problem using backward induction. Starting with the old firm, the envelope condition with respect to $k^{j,2}_{N,t+2}$

$$V^2_{1,t+2}(k^{j,2}_{N,t+2}, k^{j,2}_{O,t+2}, b^{j,2}_{t+2}) = [1 + \phi'(-d^{j,2}_{t+2})][f_1(k^{j,2}_{N,t+2}, k^{j,2}_{O,t+2}; a_{t+2}) + 1 - \delta + q_{t+2}\delta]$$

The envelope condition with respect to $k^{j,2}_{O,t+2}$

$$V^2_{2,t+2}(k^{j,2}_{N,t+2}, k^{j,2}_{O,t+2}, b^{j,2}_{t+2}) = [1 + \phi'(-d^{j,2}_{t+2})][f_2(k^{j,2}_{N,t+2}, k^{j,2}_{O,t+2}; a_{t+2}) + q_{t+2}(1-\delta)]$$

The envelope condition with respect to $b^{j,2}_{t+2}$

$$V^2_{3,t+2}(k^{j,2}_{N,t+2}, k^{j,2}_{O,t+2}, b^{j,2}_{t+2}) = -R[1 + \phi'(-d^{j,2}_{t+2})]$$

Turning to the middle-aged firm, the first order condition with respect to $k^{j,2}_{N,t+2}$

$$1+\phi'(-d^{j,1}_{t+1}) = \frac{1}{R}\mathbb{E}_{t+1}[V^2_{1,t+2}(k^{j,2}_{N,t+2}, k^{j,2}_{O,t+2}, b^{j,2}_{t+2})] + \lambda^1_{t+1}\theta_{t+1}$$
$$\implies 1 = \mathbb{E}_{t+1}\left[\frac{1+\phi'(-d^{j,2}_{t+2})}{R[1+\phi'(-d^{j,1}_{t+1}) - \lambda^1_{t+1}\theta_{t+1}]}[f_1(k^{j,2}_{N,t+2}, k^{j,2}_{O,t+2}; a_{t+2}) + 1 - \delta + q_{t+2}\delta]\right] \tag{23}$$

The first-order condition with respect to $k^{j,2}_{O,t+2}$

$$q_{t+1} + \phi'(-d^{j,1}_{t+1})q_{t+1} = \frac{1}{R}\mathbb{E}_{t+1}[V^2_{2,t+2}(k^{j,2}_{N,t+2}, k^{j,2}_{O,t+2}, b^{j,2}_{t+2})] + \lambda^{j,1}_{t+1}\theta_{t+1}q_{t+1}$$
$$\implies q_{t+1} = \mathbb{E}_{t+1}\left[\frac{1+\phi'(-d^{j,2}_{t+2})}{R[1+\phi'(-d^{j,1}_{t+1}) - \lambda^{j,1}_{t+1}\theta_{t+1}]}[f_2(k^{j,2}_{N,t+2}, k^{j,2}_{O,t+2}; a_{t+2}) + q_{t+1}(1-\delta)]\right] \tag{24}$$

The first-order condition with respect to $b^{j,2}_{t+2}$

$$1 - \phi'(-d^{j,1}_{t+1})(-1) - \frac{1}{R}\mathbb{E}_{t+1}\left[V^2_{3,t+2}(k^{j,2}_{N,t+2}, k^{j,2}_{O,t+2}, b^{j,2}_{t+2})\right] - \lambda^t_{t+1} = 0$$
$$\implies \lambda^{j,1}_{t+1} = \phi'(-d^{j,1}_{t+1}) - \mathbb{E}_{t+1}[\phi'(-d^{j,2}_{t+2})] \tag{25}$$

The envelope condition with respect to $k^{j,1}_{N,t+1}$

$$V^1_{1,t+1}(k^{j,1}_{N,t+1}, k^{j,1}_{O,t+1}, b^{j,1}_{t+1}) = [1+\phi'(-d^{j,1}_{t+1})][f_1(k^{j,1}_{N,t+1}, k^{j,1}_{O,t+1}; a_{t+1}) + 1 - \delta + q_{t+1}\delta]$$

The envelope condition with respect to $k^{j,1}_{O,t+1}$

$$V^1_{2,t+1}(k^{j,1}_{N,t+1}, k^{j,1}_{O,t+1}, b^{j,1}_{t+1}) = [1+\phi'(-d^{j,1}_{t+1})][f_2(k^{j,1}_{N,t+1}, k^{j,1}_{O,t+1}; a_{t+1}) + (1-\delta)q_{t+1}]$$

The envelope condition with respect to $b^{j,1}_{t+1}$

$$V^1_{3,t+1}(k^{j,1}_{N,t+1}, k^{j,1}_{O,t+1}, b^{j,1}_{t+1}) = -[1+\phi'(-d^{j,1}_{t+1})]R$$

Turning to the young firm, the first order condition with respect to $k^{j,1}_{N,t+1}$

$$
\begin{aligned}
1+\phi'(-d^{j,0}_t) &= \frac{1}{R}\mathbb{E}_t[V^1_{1,t+1}(k^{j,1}_{N,t+1},k^{j,1}_{O,t+1},b^{j,1}_{t+1})]+\lambda^{j,1}_t\theta_t \\
\implies 1 &= \mathbb{E}_t\left[\frac{1+\phi'(-d^{j,1}_{t+1})}{R[1+\phi'(-d^{j,0}_t)-\lambda^{j,0}_t\theta_t]}[f_1(k^{j,1}_{N,t+1},k^{j,1}_{O,t+1};a_{t+1})+1-\delta+q_{t+1}\delta]\right] \qquad (26)
\end{aligned}
$$

The first order condition with respect to $k^{j,1}_{O,t+1}$

$$
\begin{aligned}
q_t[1+\phi(-d^{j,1}_t)] &= \frac{1}{R}\mathbb{E}_t[V^1_{2,t+1}(k^t_{N,t+1},k^t_{O,t+1},b^t_{t+1})]+\lambda^t_t\theta_t q_t \\
\implies q_t &= \mathbb{E}_t\left[\frac{1+\phi'(-d^t_{t+1})}{R[1+\phi(-d^t_t)-\lambda^{j,0}_t\theta_t]}[f_2(k^t_{N,t+1},k^t_{O,t+1};a_{t+1})+(1-\delta)q_{t+1}]\right] \qquad (27)
\end{aligned}
$$

First order condition with respect to $b^{j,1}_{t+1}$

$$
\begin{aligned}
1+\phi'(-d^{j,0}_t)+\frac{1}{R}\mathbb{E}_t[V^1_{3,t+1}(k^{j,1}_{N,t+1},k^{j,1}_{O,t+1},b^{j,1}_{t+1})]-\lambda^{j,0}_t &= 0 \\
\implies \lambda^{j,0}_t = \phi'(-d^{j,0}_t)-\mathbb{E}_t[\phi'(-d^{j,1}_{t+1})] & \qquad (28)
\end{aligned}
$$

Using the incumbent characterization, aggregates can be simplified as follows.

$$
\begin{aligned}
K_{O,t} &= \mu_{t-1}k^{E,1}_{O,t}+\mu_{t-2}k^{E,2}_{O,t}+(\upsilon_{t-1}+\upsilon_{t-2})k^I_{O,t} \\
K_{N,t} &= \mu_{t-1}k^{E,1}_{N,t}+\mu_{t-2}k^{E,2}_{N,t}+(\upsilon_{t-1}+\upsilon_{t-2})k^I_{N,t} \\
Y_t &= \mu_{t-1}f(k^{E,1}_{N,t},k^{E,1}_{O,t};a_t)+\mu_{t-2}f(k^{E,2}_{N,t},k^{E,2}_{O,t};a_t)+(\upsilon_{t-1}+\upsilon_{t-2})f(k^I_{N,t},k^I_{O,t};a_t) \\
I_t &= \mu_t k^{E,1}_{N,t+1}+\mu_{t-1}[k^{E,2}_{N,t+1}-(1-\delta)k^{E,1}_{N,t}]-\mu_{t-2}(1-\delta)k^{E,2}_{N,t} \\
&\quad +(\upsilon_t+\upsilon_{t-1})k^I_{I,t+1}-(\upsilon_{t-1}+\upsilon_{t-2})(1-\delta)k^I_{N,t} \\
D^E_t &= \mu_{t-2}d^{E,2}_t+\mu_{t-1}d^{E,1}_t+\mu_t d^{E,0}_t \\
B^E_t &= \mu_{t-1}b^{E,1}_t+\mu_{t-2}b^{E,2}_t \\
\Phi_t &= \mu_{t-2}\phi(-d^{E,2}_t)+\mu_{t-1}\phi(-d^{E,1}_t)+\mu_t\phi(-d^{E,0}_t) \\
C_t &= D^E_t-\Phi_t+RB^E_t-B^E_{t+1}
\end{aligned}
$$

C Quantitative Model Characterization

Attach the collateral constraint with multiplier $\lambda_t(s)$.

$$V_t^C(s) = \max_{(d,b')\in\mathbb{R}^2,(k'_N,k'_O)\in\mathbb{R}^2_+} d - \phi(-d) + \frac{1}{R}\mathbb{E}\Big[V_{t+1}(s')\Big|z\Big] + \lambda_t(s)\Big[\theta\Big[k'_N + q_t k'_O\Big] - b\Big]$$

$$\text{s.t. } d = \pi_t(s) - c_F - k'_N - q_t k'_O + b'$$

$$\pi_t(s) = f(z, k_N, k_O; a_t) + (1-\delta_N)k_N + q_t[\delta_N k_N + (1-\delta_O)k_O] - Rb$$

The first-order condition with respect to borrowing b':

$$1 + \phi'(-d_t(s)) + \frac{1}{R}\mathbb{E}\left[\frac{\partial V_{t+1}(s')}{\partial b'}\middle| z\right] = \lambda_t(s)$$

$$\implies 1 + \phi'(-d_t(s)) + \frac{1}{R}\mathbb{E}\left[[1 - x_{t+1}(s')]\frac{\partial V^C_{t+1}(s')}{\partial b'} + x_{t+1}(s')\frac{\partial V^X_{t+1}(s')}{\partial b'}\middle| z\right] = \lambda_t(s) \tag{29}$$

The envelope conditions with respect to borrowing b:

$$\frac{\partial V_t^X(s)}{\partial b} = -R \tag{30}$$

$$\frac{\partial V_t^C(s)}{\partial b} = -R[1 + \phi'(-d_t(s))] \tag{31}$$

Combining (29), (30), and (31) pins down the multiplier on the borrowing constraint as the difference between the marginal equity issuance cost this period and the expected future marginal equity issuance cost next period.

$$\lambda_t(s) = \phi'(-d_t(s)) - \mathbb{E}\Big[[1 - x_{t+1}(z', k'_{N,t}(s), k'_{O,t}(s), b'_t(s))]\phi'(-d_{t+1}(z', k'_{N,t}(s), k'_{O,t}(s), b'_t(s)))\Big| z\Big] \tag{32}$$

The first-order condition with respect to new capital k'_N:

$$1 + \phi'(-d_t(s)) = \lambda_t(s)\theta + \frac{1}{R}\mathbb{E}\left[\frac{\partial V_{t+1}(s')}{\partial k'_N}\middle| z\right]$$

$$= \lambda_t(s)\theta + \frac{1}{R}\mathbb{E}\left[[1 - x_{t+1}(s')]\frac{\partial V^C_{t+1}(s')}{\partial k'_N} + x_{t+1}(s')\frac{\partial V^X_{t+1}(s')}{\partial k'_N}\middle| z\right] \tag{33}$$

The envelope conditions with respect to new capital k_N:

$$\frac{\partial V^X_t(s)}{\partial k'_N} = f_2(z, k_N, k_O; a_t) + (1 - \delta_N) + q_t\delta_N \tag{34}$$

$$\frac{\partial V^C_t(s)}{\partial k_N} = [f_2(z, k_N, k_O; a_t) + (1 - \delta_N) + q_t\delta_N][1 + \phi'(-d_t(s)] \tag{35}$$

Combining (32), (33), (34), and (35)

$$1 + \phi'(-d_t(s)) = \lambda_t(s)\theta + \frac{1}{R}\mathbb{E}\bigg[\Big(f_2(z', k'_{N,t}(s), k'_{O,t}(s); a_{t+1}) + 1 - \delta_N + q_{t+1}\delta_N\Big) \tag{36}$$

$$* \Big(1 + \phi'(-d_{t+1}(z', k'_{N,t}(s), k'_{O,t}(s))\big[1 - x_{t+1}(z', k'_{N,t}(s), k'_{O,t}(s))\big]\Big)\bigg| z\bigg] \tag{37}$$

$$1 + (1-\theta)\phi'(-d_t(s)) = \theta\mathbb{E}\Big[[1 - x_{t+1}(z', k'_{N,t}(s), k'_{O,t}(s), b'_t(s))]\phi'(-d_{t+1}(z', k'_{N,t}(s), k'_{O,t}(s), b'_t(s)))\Big| z\Big] \tag{38}$$

$$+ \frac{1}{R}\mathbb{E}\bigg[\Big(f_2(z', k'_{N,t}(s), k'_{O,t}(s); a_{t+1}) + 1 - \delta_N + q_{t+1}\delta_N\Big) \tag{39}$$

$$* \Big(1 + \phi'(-d_{t+1}(z', k'_{N,t}(s), k'_{O,t}(s))\big[1 - x_{t+1}(z', k'_{N,t}(s), k'_{O,t}(s))\big]\Big)\bigg| z\bigg] \tag{40}$$

FOC wrt k'_O

$$q_t[1 + \phi'(-d_t(s))] = \lambda_t(s)\theta q_t + \frac{1}{R}\mathbb{E}\left[\frac{\partial V_{t+1}(s')}{\partial k'_O}\middle| z\right]$$