Econ712 - Handout 2

1 Moderately timed review

1.1 Modeling

- Environment: Agents; Preferences/Payoffs; Technology; Information
- Equilibrium

1.2 Class material

- Consumption - labour decision
- Planner's allocation vs competitive equilibrium allocation

2 Dynamics and Difference equations

- Macro deals with problems that are dynamic in nature
- In discrete time, equilibrium characterized by difference equations
- Solution/characterization: Linearization; Phase Diagrams; Numerical

2.1 Some bivariate linear difference equation

Consider the following system

$$\Omega_{t+1} = (1-\delta)\Omega_t + L_t^c$$
$$L_{t+1}^c = (1+\delta+\beta)L_t^c + \gamma\Omega_t - \theta_0$$

Here $\delta \in (0,1), \beta \in (0,1), \gamma \geq 0$. Illustrate the system dynamics using a phase diagram with Ω_t on the horizontal axis and L_t^c on the vertical axis. Analyze the regions where $\Delta\Omega_t > 0, \Delta\Omega_t = 0, \Delta\Omega_t < 0$, and do the same for L_t^c. Is there a steady state $(\bar{\Omega}, \bar{L}^c)$ and saddle path?

2.2 Finite consumption - saving

Consider an agent that lives for T periods with time seperable utility. She ranks consumption each period according to $u(C)$, and discounts future consumption geometrically at rate β. Each period she is endowed with $w > 0$ units of consumption good. She has access to a perfect storage technology, whereby 1 unit of good saved today will give her 1 unit of good tomorrow. Assume $u(C)$ is strictly increasing, strictly concave, and continuously differentiable, with $\lim_{c\to 0} u'(c) = \infty$ and $\lim_{c\to\infty} u'(c) = 0$. In period 0, the agent manages to find treasures valued at S.

1. Derive the difference equations that characterize the agent's consumption and savings decisions.
2. Let $T = 23$, $w = 1$, $u(c) = \log(c)$, $\beta = 0.99$, $S = 34$. Solve numerically for the optimal consumption and savings decisions.

3 Setting up a model

State the Consumer Problem and define the Competitive Equilibrium for the following:

Consider an overlapping generations economy of 3-period-lived agents. Denote these periods as *young, mid, old.* At each date $t \geq 1$, a measure 1 of new young agents enter the economy, each endowed with w_1 units of the consumption good when young, w_2 units when mid, and w_3 units when old. The consumption good is non-storable. Consumption preference is described by $\ln c_t^t + \ln c_{t+1}^t + \ln c_{t+2}^t$. At time $t = 1$, there is an unit measure of old agents, each endowed with w_3 units of the consumption good; and an unit measure of mid agents, each endowed with w_2 units of the consumption good at $t = 1$ and w_3 units at $t = 2$. Additionally, each inital old agent is endowed with 1 unit of fiat currency.