ECON 712A: Macroeconomic Theory - Section Handout 3

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Correction from Handout 1:

- Duong's office hours are Tuesday 10:30 AM - 11:45 AM in 6439 Social Science.
- Alex's office hours are Thursday at 2:15 PM - 3:30 PM in 6473 Social Science.

Content Review

- Labor/leisure notes:
  - The **Lucas critique** argues that relationships observed in historical data may not be invariant to changes in government policy. In lecture, we saw an example of how the Lucas critique applies to a linear regression of labor supply on productivity.
  - The **First Welfare Theorem** states that under appropriate assumptions, a competitive equilibrium is pareto optimal.
  - The **Second Welfare Theorem** states that under appropriate assumptions, any pareto optimal allocation can be achieved as a competitive equilibrium with appropriate lumpsum taxes and transfers.
- Kehoe notes:
  - Similar OG framework as introduction notes.
  - Old agents have endowment $w_2 > 0$.
  - **Discount factor** $\beta \in [0, 1]$ in utility function: $U(c_t^t, c_{t+1}^t) = \ln(c_t^t) + \beta \ln(c_{t+1}^t)$
  - **Inside vs. outside money**: $M_{t+1}^t \in \mathbb{R}$
  - **Intertemporal budget constraint**: inside money allows us to consolidate budget constraints: $p_t c_t^t + p_{t+1} c_{t+1}^t = p_t w_1 + p_{t+1} w_2$
  - **Excess demand** is demand minus endowment: $x_1(p_t, p_{t+1}) = c_t^t - w_1$ and $x_2(p_t, p_{t+1}) = c_t^{t+1} - w_2$.
  - **Market clearing** in excess demand space implies that $\frac{x_2(p_{t-1}, p_t)}{x_1(p_t, p_{t+1})} = -1$.
  - **Offer curve** is the locus of optimal excess demands as the price ratio varies between 0 and ∞: $\frac{x_2(p_t, p_{t+1})}{x_1(p_t, p_{t+1}} = -\frac{p_t}{p_{t+1}}$.
  - Walk through the top panel of Figure L3.2.

A couple things from problem set 1

- Where do **prices** show up?
  - The planner problem is to maximize present value of utility subject to resource constraint.
  - No prices in the planner problem.
  - Competitive equilibrium: Taking prices as given, households optimize subject to budget constraint and firms optimize subject to production technology. Apply market clearing to find prices.
  - No prices appear in market clearing conditions.
- In dynamic optimization problems, there are **control variables** and **state variables**.
  - Each period, agents wake up, look at their state variables (e.g., wealth today), make decision about their control variables (e.g., consumption today and wealth tomorrow), and then go back to sleep.
  - We'll talk a lot more about control and state variables in second quarter.
- **Discount factor**, **geometric discounting**, **interest rates**, and **present value**:
  - Discount factor $\beta \in (0,1)$ is a model primitive that captures impatience. Agents would prefer consuming now.
  - Geometrically discounting of steam of future consumption: $U(c) = \sum_{t=0}^{\infty} \beta^t u(c_t)$
  - "Geometric" because of the geometric series: $\sum_{t=0}^{\infty} \beta^t = \frac{1}{1-\beta}$.
  - Why geometric discounting? Time consistent. Other ways to discount include hyperbolic discounting (time inconsistent; outside scope of this class).
  - Rate of return r vs. gross interest rates R: $r + 1 = R$.
  - Discount factor is a primitive (exogenous) and risk-free interest rate is a price (endogenous), but they are often related in deterministic representative agent models: $\beta = \frac{1}{R^f}$.
  - Using gross interest rate R, present value of payoff x in t periods: $PV(x) = R^{-t} x_t$.
  - Using gross interest rate R, present value of stream of payoffs $y = \{y_t\}_{t=0}^{T}$: $PV(y) = \sum_{t=0}^{T} R^{-t} y_t$.
- Once we start going through "Government Policy in an OG Environment" in lecture, take a look at the solution to problem 5.

Trade Offer Curves

Plot the trade offer curves for the following utility functions where the endowment is (w_1, w_2) for goods 1 and 2, respectively.

(a) $U = 10c_1 - 4c_1^2 + 4c_2 - c_2^2, (w_1, w_2) = (0, 2)$

(b) $U = \min\{2c_1 + c_2, c_1 + 2c_2\}, (w_1, w_2) = (1, 0)$

(c) $U = \min\{2c_1 + c_2, c_1 + 2c_2\}, (w_1, w_2) = (1, 10)$

Gale's Pure Exchange Equilibrium of Dynamic Economic Models

Consider the following overlapping generations problem.[1] In each period t a new generation of 2 period lived households are born, and each generation has a unit measure. Each generation is endowed with $w_1 = 0$ in youth and $w_2 = 2$ in old age of nonstorable consumption goods. In each period the agents can buy or issue money $M_{t+1}^t \in \mathbb{R}$, to some outside agent. We assume that the market for money always clears. The utility function of a household of generation t is quadratic:[2]

$$U\left(c_t^t, c_{t+1}^t\right) = 10c_t^t - 4(c_t^t)^2 + 4c_{t+1}^t - (c_{t+1}^t)^2 \tag{1}$$

where $\left(c_t^t, c_{t+1}^t\right)$ is consumption of a household of generation t in youth (i.e. in period t) and old age (i.e in period $t+1$). We ignore the initial old.

1. Write down the problem of generation t with a consolidated budget constraint.
2. Find the Euler equation (intertemporal optimality condition) and eliminate prices from your expression.
3. Use the market clearing condition for goods to find the two steady state consumption allocations for the young and old in equilibrium.
4. Find all 2-period cyclical*(!)* competitive equilibrium (e.g. $c_t^t = c_{t+2}^{t+2}, c_{t+1}^t = c_{t+3}^{t+2}\ \forall t$).
5. In the excess demand space, draw the offer curve and market clearing. Show that, depending on the initial price, we can get an equilibrium where excess demand 'jump' around the steady state, instead of smoothly converging.

[1] Gale, D. (1973). *Pure exchange equilibrium of dynamic economic models.* Journal of Economic Theory, 6(1), 12-36.

[2] Quadratic utility is still used occasionally, e.g. Cochrane, J. (2014) *A Mean-Variance Benchmark For Intertemporal Portfolio Theory*, Journal of Finance.