Econ712 - Handout 4

1 Competitive equilibrium with social security

1.1 Environment

- Demographics: Discrete time, 2 period lived households, population growth n
- Technology:
  - Endowments (w_1, w_2) of non-storable consumption goods in youth and old age
  - Commitment technology that allows trade to take place across generations
- Prefences:
  - Initial old: $u(c_1^0)$
  - Generation t: $u(c_t^t) + \beta u(c_{t+1}^t)$
  - Assume $\lim_{c \to 0} u'(c) = \infty$
  - Assume $\beta u'(w_2) > u'(w_1)$
- Social security system: the young pay lumpsum taxes $\tau \in [0, w_1)$ and recieve transfers b when old

1.2 Equilibrium

1. Set up the household's optimization problem
2. What is the government's budget constraint?
3. Define a Competitive Equilibrium with Social Security

1.3 Characterization

1. Argue that autarky is the only equilibrium
2. Denote the lifetime utility under autarky by

$$V(\tau) = u(w_1 - \tau) + \beta u(w_2 + \tau(1+n))$$

   Starting from no social security system ($\tau = 0$), can a marginal increase in τ increase lifetime utility? Does the introduction of the social security system pareto improve over (strict) autarky?
3. What is the optimal social security tax and transfers?

2 Computing Life Cycle model

Consider a more general life cycle model with J period-lived agents. Newly born agents are endowed with no capital, but can subsequently save in capital which they can rent to firms at rate r. A worker of age j supplies labour $l_j \in [0,1]$ and pays proportional security tax on her labour income $\tau w e_j l_j$, where e_j is an age efficiency profile. For $j \geq J^R$, the worker retires ($l_j = 0$) and receives pension benefits b.

Preferences are given by

$$\sum_{j=1}^{J} \beta^{j-1} u(c_j, l_j) = \sum_{j=1}^{J} \beta^{j-1} \left(\frac{c_j^{\gamma}(1-l_j)^{1-\gamma}}{1-\sigma} \right)^{1-\sigma}$$

We will go through the steps to compute a stationary equilibrium of this economy.

2.1 Firm block

The production tech is $Y = F(K,L) = K^{\alpha} L^{1-\alpha}$. Capital depreciates at δ. Labor and capital markets are perfectly competitive, so that $w = F_2(K,L)$ and $r = F_1(K,L) - \delta$

2.2 Household block

The problem of the household:

$$\max \sum_{j=1}^{J} \beta^{j-1} \left(\frac{c_j^{\gamma}(1-l_j)^{1-\gamma}}{1-\sigma} \right)^{1-\sigma} \qquad s.t.$$

$$c_j + k_{j+1} = (1-\tau) w e_j l_j + (1+r) k_j \quad j = 1, \ldots, J^{R-1}$$
$$c_j + k_{j+1} = b + (1+r) k_j \quad j = J^R, \ldots, J$$

First note that the optimal l_j (if unconstrained) is given by

$$l_j = \frac{\gamma(1-\tau) e_j w - (1-\gamma)\left[(1+r)k_j - k_{j+1}\right]}{(1-\tau) e_j w}$$

We could solve this problem in a variety of ways:

1. Euler Equation approach:

   (a) Guessing that $k_{j+1} \geq 0$ will not bind, the consumption Euler equation is

   $$u_1(c_j, l_j) = \beta u_1(c_{j+1}, l_{j+1})\left[1+r\right]$$

   (b) With the boundary conditions $k_1 = k_{J+1} = 0$, we can solve this either using the shooting method or by solving for the policy functions $c_j(k_j)$ and $k_{j+1}(k_j)$ (recall handout 2)

2. Value Function approach:

(a) Denote

$$V_j(k) = \max_{c,l,k'} \left(\frac{c^\gamma (1-l)^{1-\gamma}}{1-\sigma} \right)^{1-\sigma} + \beta V_{j+1}(k') \quad s.t.$$

$$\begin{cases} c + k' = b + (1+r)k & j = J^R, \dots, J \\ c + k' = (1-\tau) w e_j l + (1+r)k & j = 1, \dots, J^{R-1} \end{cases}$$

Then $V_1(0)$ is the optimal lifetime utility of the agent

(b) Let $V_{J+1}(k) = 0$. Solve numerically for V_J, then V_{J-1}, and so on

(c) A byproduct of this are the policy functions $c_j(k_j)$ and $k_{j+1}(k_j)$

2.3 Aggregation

Aggregate K supply and L supply can be found be summing up capital and labour supply across generations, with appropriate generation weights. With growth rate n, the relative size of each cohort is given by $\psi_{i+1} = (1+n)^{-1}\psi_i$ with $\psi_1 = \tilde{\psi}$. Then

$$K = \sum_{j=1}^{J} \psi_j k_j$$

$$L = \sum_{j=1}^{J^R - 1} \psi_j e_j l_j$$

2.4 Gov block

The gov budget constraint is

$$b = \frac{\tau w L}{\sum_{j=J^R}^{J} \psi_j}$$

2.5 Equilibrium and Algorithm

Finding a stationary equilibrium entails finding allocations $\{c_j, l_j, k_{j+1}\}$, prices (r, w), and policies (τ, b) such that

1. Household optimization is satisfied, given prices and policies
2. Firm's optimization satisfied
3. Markets clear: Capital supply = Capital demand; Labour supply = Labour demand; Goods supply = Goods demand
4. Gov BC is satisfied

The algorithm is as follows: Given some τ:

1. Outer block - Searching for prices: Guess w, r. Calculate the implied L and K supply, and b

   (a) Inner block: Solve for household optimal allocations as described above

(b) Calculate aggregate L and K demand

2. If supply = demand, stop. Else update guess of w, r

Alternatively, one could guess L and K supply, which gives an implied w, r.