ECON 712A: Handout 5 - Solution[1]

Instructor: Dean Corbae
TAs: Duong Dang and Alex von Hafften

Content Review

Ricardian Equivalence

- The government can raise government expenditures today by either

  (1) increasing taxes today ($\Delta g_t = \Delta T_t$) or

  (2) borrowing today and increasing taxes in the future ($\Delta g_t = q_t \Delta B_{t+1}$).

  Either way of financing government expenditure is feasible and consistent with government consolidated budget constraint. The government can also finance an increase in expenditure by any feasible linear combination of (1) and (2).

- Explain why "the timing of taxes does not matter".

  The households are indifferent between having an increase in taxes today or at some point in the future as a result of an increase in government borrowing today.

  In the simple two period model, we can show that $\Delta g_1 = q_1 \Delta B_2 = q_1 \Delta T_2$. The household is indifferent because the change in her intertemporal budget constraint is the same under the two taxation strategies: $\Delta(c_1 + q_1 c_2) = \Delta((\omega_1 - T_1) + q_1(\omega_2 - T_2)) = -\Delta g_1$.

- What assumptions underpin Ricardian Equivalence?

  We talked about two assumptions underpinning Ricardian Equivalence:

  - Perfect financial markets (e.g., no borrowing constraints)

  - Lump-sum taxes.

  We have seen one example of optimal taxation with distortionary taxes in class (see "Intro to Government Commitment Problems"). You will see more examples in fourth quarter.

- How Ricardian Equivalence can hold in OLG models?

  This week, we argued that Ricardian Equivalence can hold in OLG model with the bequest motive (i.e. agents care about their kids).

[1]Based on handout by Katya Kazakova.

Government Commitment Problem

- Set up from section 3.4 of lecture notes:

  - Households invest x in productive technology with return $(1-\tau)R$

  - Households save m in pillow technology with return 1.

  - Because returns are linear and exogenous, households put their entire endowment in pillow technology if $1 \geq (1-\tau)R$.

  - Note that we may have both corner and internal solutions depending on parameters. [Don't blindly take FOCs, especially with linear returns.]

- What does "Big X, small x" mean?

  If we assume there is a continuum of households, each households is atomless, so their choice of x does not affect the aggregate X. In practice, we take first order conditions with respect to x and treat X as a constant.

- What does the consistency requirement mean?

  Consistency (given symmetry) requires that $x = X$. In practice, after we take first order conditions, we set $x = X$.

- Which agent (i.e. government or households) acts first in a Ramsay equilibrium (i.e. equilibrium with commitment)?

  In Ramsey equilibria, the government acts first and chooses taxes, then households make their decisions. We use backward induction to solve the two period game. Start with decision-making by households and then find taxes that maximize welfare given households' optimal decisions. Crucially, the government internalizes its influence on households.

- Which agent (government or households) acts first in a Nash equilibrium (i.e. equilibrium without commitment)?

  In Nash equilibria, households make consumption and savings decisions first, then the government chooses taxes. The government does not take into account its influence on households. In these equilibria, governments tends to 'overtax' investment in the second period. Households anticipate this overtaxing and choose low investment in the first period. As a result, equilibria with commitment are preferred to the equilibria without commitment.

- Why does the government overtax investment in the equilibrium without no commitment ("dynamic inconsistency")?

  The reason why the government 'overtaxes' investment is because investment is sunk at period two. We have already seen that inelastic goods are taxed at higher rates (example with labor supply). The government does not internalize its influence on households and tends to tax irreversible investment as much as possible.

- What is a Laffer curve? What is the optimal choice of distortionary tax and how does it relate to a Laffer curve?

  Laffer curve is a tax revenue curve. If the government wants to maximize its tax revenue then the optimal tax rate would be the one which maximizes a Laffer curve.

  In an example from class, the government's tax revenue is $g = \tau RX$ which is increasing in τ until $\tau = \frac{R-1}{R}$ and drops to zero for $\tau > \frac{R-1}{R}$. Notice that by maximizing households' welfare, the government optimally chooses the tax rate which corresponds to maximum of tax revenues (highest point on a Laffer curve).

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Midterm Exam

Instructions: You have 2.5 hours to complete this out-of-class (during Covid-19) exam and 15 minutes to submit. You may only submit your answers provided you abide by the following rules. The exam is open notes. Your answers on this exam must be only your own work. Copying someone else's answers, letting someone copy mine, or discussing/sharing these questions with anyone (in or out of the class, online or offline) during or after the exam counts as cheating, and the penalty for cheating is a 0 on the exam which cannot be dropped.

In this midterm, we consider the possibility of government commitment problems. We will consider several different assumptions about the commitment technology. The main body of the paper considers only 2 period environments and then briefly consider what would happen in a more general finite period problem.

Environment

- There are two periods $t = 1$ and $t = 2$
- Population: There is a unit measure of households and a big government.
- Technology:
  - Each household is endowed with $w < 1$ units of a consumption good in period $t = 1$ and 1 unit of time in period $t = 2$.
  - There is a productive storage technology which transforms k units of the endowment at $t = 1$ into Rk units of the good in period $t = 2$ where $R > 1$.
  - Households can divide their 1 unit of time in period $t = 2$ to work n or leisure $1 - n$. If they work n units of time at $t = 2$, they produce n units of the consumption good (i.e. their production function is $y = n$).
- Preferences: A household's utility function is given by $U(c_1, c_2, n) = \ln(c_1 + c_2) + \ln(1 - n)$ where $c_t \geq 0$ denotes consumption in period $t = 1$ or 2. Note that $t = 1$ and $t = 2$ consumption goods are perfect substitutes (i.e. it doesn't matter when a household consumes) and that consumption in any period cannot be negative.

- There is an exogenous amount of government expenditure g in $t = 2$ which does not enter the household utility function (i.e. g is taken as given from outside the model). Assume that $g > Rw$ so that even if all endowment is put into the productive technology, it does not produce enough goods to cover government expenditure.

- Additional Parametric Assumptions (if there are additional parametric assumptions you feel should be added, state them and why):

  - A1: $(1 + g - Rw)/2 \in (0, 1)$.

Case 1 - Planner's problem

1. (7.5 points) State the planner's problem if she weights all households equally (don't forget there is a resource constraint for each period).

   Answer:

$$\max_{c_1, c_2, n, k} U(c_1, c_2, n)$$

   s.t.

$$\begin{aligned} c_1 + k &= w \\ c_2 + g &= n + Rk \end{aligned}$$

2. (12.5 points) What is the planner's allocation?

   Answer:

$$\max_{n,k} \ln(w - k + n + Rk - g) + \ln(1 - n)$$

   Before taking derivatives, note that since $R > 1$, the planner will choose $k = w$ (respecting the non-negativity constraint on c_1). Then the foc for n is:

$$\frac{1}{(n + Rw - g)} = \frac{1}{1 - n} \Longleftrightarrow 1 - n = n + Rw - g \Longrightarrow n = \frac{1 + g - Rw}{2}.$$

   Thus the allocation is $c_1^* = 0$, $k^* = w, n^* = (1 + g - Rw)/2$, and $c_2^* = (1 - g + Rw)/2$. Assumption A1 guarantees $n^* \in (0, 1)$.

Case 2 - Decentralized Equilibrium with commitment (Ramsey)

Now consider a competitive equilibrium where the government taxes labor earnings at proportional rate $\tau \in [0, 1 - w]$ (i.e. if the household works n hours, the government receives τn in revenue) and taxes storage returns at proportional rate $\delta \in [0, 1]$ (i.e. if the household stores k units of its $t = 1$ endowment, the government receives δRk in revenue). Assume that the government chooses τ and δ before the household chooses its storage decision k in period $t = 1$ and labor supply decision n in period $t = 2$.

3. (2.5 Points) State the government budget constraint.

   Answer.

$$g = \tau n + \delta R k$$

4. (7.5 Points) For a given τ and δ, state the household problem.

   Answer.

$$\max_{c_1, c_2, n, k} U(c_1, c_2, n)$$

$$\begin{aligned} c_1 + k &= w \\ c_2 &= (1-\tau)n + (1-\delta)Rk \end{aligned}$$

5. (15 Points) Solve the household problem (assuming that if the household is indifferent between whether to store or not, it stores).

Answer. If $(1-\delta)R \geq 1$ or $\delta \leq \delta^r \equiv (R-1)/R$, then $\overline{k}^r = w$ and $\overline{c}_1^r = 0$ otherwise $\underline{k}^r = 0$ and $\underline{c}_2^r = w$. Assuming $\delta \leq \delta^r$, then the remaining labor choice must satisfy:

$$\max_n \ln((1-\tau)n + (1-\delta)Rw) + \ln(1-n)$$

In that case, the foc for n is:

$$\begin{aligned} \frac{1-\tau}{((1-\tau)n + (1-\delta)Rw)} &= \frac{1}{1-n} \iff (1-\tau)(1-n) = (1-\tau)n + (1-\delta)Rw \\ &\iff (1-n) = n + \frac{(1-\delta)Rw}{(1-\tau)} \implies \overline{n}^r = \frac{1-(1-\delta)Rw/(1-\tau)}{2}. \end{aligned}$$

Then to keep $\overline{n}^r \geq 0$, we need $1 - w \geq \tau$.

If $\delta > \delta^r$, then the problem is

$$\max_n \ln(w + (1-\tau)n) + \ln(1-n)$$

foc for n is

$$\frac{1-\tau}{w + (1-\tau)n} = \frac{1}{1-n} \implies \underline{n}^r = \frac{1 - w/(1-\tau)}{2}.$$

Then to keep $\underline{n}^r \geq 0$ we need $1 - w \geq \tau$ (the same as above).

6. (5 Points) Taking the decision rules $n^r(\tau, \delta)$ and $k^r(\tau, \delta)$ in question 5 as given, state the government's problem.

   Answer.

$$\max_{\tau, \delta} U(c_1^r, c_2^r, n^r)$$

$$\begin{aligned} c_1^r + k^r &= w \\ c_2^r + g &= n^r + Rk^r \\ g &= \tau n^r + \delta R k^r \end{aligned}$$

7. (15 Points) Provide one equation in one unknown which characterizes a solution to the government's problem (you needn't provide a closed form solution). Explain your logic. Hint: Think about what the planner would do.

Answer. If the government chooses $\delta > \delta^r$, then $\underline{k}^r(\tau, \delta) = 0$ and since the government raises no revenue from capital taxation, all revenue must be raised from distortionary taxation, which is inefficient (i.e. not what the planner would do). Instead if the government chooses $\delta \leq \delta^r$, then $\overline{k}^r(\tau, \delta) = w$ and

$$\overline{n}^r = \frac{1}{2} - \frac{(1-\delta)Rw}{2(1-\tau)}.$$

Then since

$$\frac{\partial n}{\partial \tau} = -\frac{(1-\delta)Rw}{2(1-\tau)^2} < 0,$$

taxing labor is distortionary while k is set in place so that δ is not distortionary, the government should choose $\delta = \delta^r$ and then set τ to raise the remaining revenue to satisfy the government budget constraint. That is

$$g - (R-1)w = \tau n^r(\tau, \delta) \Longleftrightarrow \tau^r = \frac{2\left[g-(R-1)w\right](1-\tau^r)}{(1-\tau^r)-(1-\delta^r)Rw}$$

which is one equation in one unknown τ^r.

Case 3 - Decentralized Equilibrium without commitment

Consider the same competitive equilibrium as in Case 2 but where the government chooses taxes (τ, δ) after households make their storage choice but before they make their labor supply choice.

8. (15 Points) Given k and (τ, δ), what is the household's decision rule $n^n(\tau, \delta, k)$ for labor supply at $t = 2$?

Answer. Given the household's budget constraint, the household choice of n implies a choice of c_2. Specifically, the hh solves:

$$\max_n \ln\left(w - k + (1-\tau)n + (1-\delta)Rk\right) + \ln(1-n)$$

The foc is

$$\begin{aligned}
\frac{(1-\tau)}{(w-k+(1-\tau)n+(1-\delta)Rk)} &= \frac{1}{1-n} \Longleftrightarrow \\
(1-\tau)(1-n) &= (w-k+(1-\tau)n+(1-\delta)Rk) \Longleftrightarrow \\
n^n &= \frac{(1-\tau)-w+k-(1-\delta)Rk}{2(1-\tau)} \Longleftrightarrow \\
n^n &= \frac{1}{2} - \frac{w-k+(1-\delta)Rk}{2(1-\tau)}
\end{aligned}$$

In order for $n^n \geq 0$, we must have

$$\begin{aligned}\frac{1}{2} - \frac{w - k + (1-\delta)Rk}{2(1-\tau)} &\geq 0 \Longleftrightarrow \\ (1-\tau) &\geq w - k + (1-\delta)Rk \Longleftrightarrow \\ 1 - w + k - (1-\delta)Rk &\geq \tau \qquad (1)\end{aligned}$$

Hence, if τ violates (1), then $n^n = 0$.

9. (10 Points) Given k and household decision rule $n^n(\tau, \delta, k)$, what is the optimal choice of (τ, δ) by the government? Again, provide one equation in one unknown which characterizes a solution to the government's problem (you needn't provide a closed form solution). Explain your logic.

Answer. Since k is in place and taxing labor supply is distortionary, the government will set $\delta^n = 1$. Notice further from (1) that setting $\delta^n = 1$ provides the most slackness in the constraint on the choice of τ^n. In this case, τ^n must satisfy the government budget constraint

$$\begin{aligned}g &= \tau n^n(\tau, 1, k) + Rk \Longleftrightarrow \\ g &= \tau^n \left(\frac{1}{2} - \frac{w-k}{2(1-\tau^n)} \right) + Rk.\end{aligned}$$

10. (5 Points) Given government decision rules $(\tau^n(k), \delta^n(k))$, what is the household's decision rule for capital at $t = 1$?

Answer. Knowing the government will choose $\delta^n(k) = 1$, they set $k^n = 0$.

11. (5 Points) Suppose this 2 period problem is repeated a finite number of times. Can the Ramsey equilibrium be implemented without commitment?

Answer. No. By backward induction, in the last period, the government will choose to overtax capital, so households choose $k = 0$. Given the last period is static, the $T - 1$ game is also static.