

1 Diamond OG - Overview

- OG with capital accumulation
- Similar steps to solve as previous OG:
 - Planner problem: FOCs lead to system of difference equation - capital is dynamic
 - Competitive equilibrium:
 - * Set up agents (household + firm) problems. Derive agents policies (optimization/FOCs)
 - * Solve for prices to clear markets under agents' policies
 - * Equilibrium allocations entail system of difference equation - capital is dynamic

Things to think about: Have we seen this environment somewhere prior to this week? Does the introduction of money in this environment matter?

2 Variation on the baseline Diamond OG

Take the environment as in class, with some changes in **bold**:

- Pop: Two period lived agents, **population growth** $N_{t+1} = (1 + n)N_t$, with $N_0 = 1$
 - Endowments: Agents have 1 unit of labour time when young
 - Tech:
 - Production $Y_t = F(K_t, L_t) = K_t^\alpha L_t^{1-\alpha}$
 - Capital accumulation: $K_{t+1} = I_t + (1 - \delta)K_t$. **Allow for general** $\delta \in [0, 1]$
 - Consumption good and capital good can be exchanged 1 for 1
 - \bar{K}_1 given for initial old
 - Pref: $\ln c_t^t + \beta \ln c_{t+1}^t$
1. Setup the planner's problem and derive the system of difference equation that characterize the planner's solution, given that the planner weighs each agent born in time t by $\gamma^t/(1+n)^t$. What is the steady state capital stock under the planner solution?
 2. Consider the decentralized competitive equilibrium of the above environment: Competitive firms rent capital from the old at r_t and hire labour from the young at w_t to maximize profits. Profits are then transferred to the old.
 - (a) What are the conditions that characterize firm demand for capital and labour? Are there any profits?
 - (b) Setup the household problem and derive the household policies
 - (c) Characterize the competitive equilibrium
 - (d) Is the stationary capital stock in the competitive equilibrium efficient?