ECON 712A: Handout 7

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Administrative Information

- Please fill out early TA evaluations (close on October 23rd).
- Midterm at 7:15 PM on November 1st. Bring pens/pencils. No need for a calculator. We will provide blue books for answers and scratch paper.

Content Review - Idiosyncratic Uncertainty

- **"Law of Large Numbers"**
  - Two-period OG model with unit measure of each generation.
  - Preferences: $U(c_t^t, c_{t+1}^t) = u(c_t^t) + u(c_{t+1}^t)$.s
  - Olds are unemployed (observable) with idiosyncratic probability π.
  - By "LLN", the mass of unemployed agents is also π.
  - Planners objective: $u(c_t^t) + \pi u(c_{t+1}^{u,t}) + (1-\pi)u(c_{t+1}^{e,t})$
  - Here, π is the mass of unemployed old agents and $1-\pi$ is the mass of employed old agents.
  - Ex-ante HH objective:

$$\pi[u(c_t^t) + u(c_{t+1}^{u,t})] + (1-\pi)[u(c_t^t) + u(c_{t+1}^{e,t})] = u(c_t^t) + \pi u(c_{t+1}^{u,t}) + (1-\pi)u(c_{t+1}^{e,t})$$

  - Here, π is probability of being unemployed and $1-\pi$ is probability of being employed.
- **Arrow-Debreu securities** are one-period assets that pay \$1 in a single future state.
- A **complete market** has assets that span the state space i.e. the number of assets with distinct payoffs is equal or greater than the number of states.
  - A market with an Arrow-Debreu security for each future state is complete.
  - Complete markets result in risk-sharing, i.e. consumption allocations depend on aggregate consumption not realization of idiosyncratic risk.

Next Steps - Incentive Compatibility

- What if the planner cannot observe idiosyncratic risk realizations?
- We solve the planner problem subject to incentive compatibility constraints.
- Incentive compatibility constraints results in allocations such that telling the truth gives higher utility than lying.

Private Commitment with Idiosyncratic Risk[1]

Consider the possibility of private commitment problems in a two period economy with a large number of agents who are subject to idiosyncratic income shocks. We will consider several different assumptions about the commitment technology.

Environment

- Population: There is a unit measure of type $i = B$ agents and a unit measure of type $i = S$ agents.
- Technology:
  - Type S agents have w_1 units of the consumption good at time $t = 1$ and 0 at $t = 2$.
  - Type B agents have 0 units of the consumption good at time $t = 1$. At time t=2, they have w_2^H in state $\theta = H$, which occurs with probability p, and w_2^L in state $\theta = L$, which occurs with probability $1 - p$.
- Preferences:
  - Let $c_t^i(\theta)$ denote consumption of a type i agent in period t in state θ
  - Type S agents have log utility $u(c_t^S(\theta)) = \ln(c_t^S(\theta))$ for both periods.
  - Type B agents have log utility $u(c_t^B(\theta)) = \ln(c_t^B(\theta))$ in period 1, but linear utility $u(c_t^B(\theta)) = c_t^B(\theta)$ in period 2.
  - Note: the state θ only matters in period 2.

Case 1 - Planner's problem with commitment

Assume agents can commit to all trades. State and solve the planner's problem given that she gives equal weight to every agent in the economy in the following steps:

1. State the planner's problem.
2. What is the planner's allocation? Hint: with linear preferences there may be many allocations which satisfy the planner's solution.

Case 2 - Bonds with commitment

Now consider a competitive equilibrium with a **non-state-contingent** private bond market. A person of type i can borrow or save in period $t = 1$ in a noncontingent bond a_{t+1}^i at price q. Specifically, if an agent of type i chooses $a_2^i > 0$, then she gives up qa_2^i goods at $t = 1$ and receives a_2^i goods at $t = 2$ and if she chooses $a_2^i < 0$, then she receives qa_2^i goods at $t = 1$ and must pay back a_2^i goods at $t = 2$. Note that neither prices nor assets depend on H or L.

[1] Based on the midterm from 2019.

3. Which type agent is a natural candidate for a borrower versus a saver and why? What are the optimization problems of each type agent?

4. What are the asset and goods market clearing conditions? Define a competitive equilibrium in this environment.

5. Solve for a competitive equilibrium.

6. Does the competitive equilibrium implement the efficient allocation? Why or why not?

Case 3 - Bonds without commitment

Now suppose that a borrower can choose not to repay their debt, but if so, she incurs a utility loss K_θ that is state dependent. That is, if the borrower defaults in state $\theta = L$, then the utility cost is K_L. If the borrower defaults in state $\theta = H$, then the utility cost is K_H. Assume that $K_H > 1 > K_L$ and that $w_L > K_L$. The parameterization $K_H > K_L$ is meant to capture that there may be more stigma attached to a wealthy person who defaults.

7. What conditions need to be satisfied for the borrower to choose **not** to default in each of the states $\theta = L$ and $\theta = H$? Can the competitive allocation **with commitment** found in Case 2 be implemented under no commitment? Hint: What are the individual rationality constraints here?

8. Re-solve for a new competitive equilibrium under which the borrower never defaults. For notation, denote the price of the bond in this new equilibrium with Q. Provide intuition on how this new price Q compares to the price q found in Case 2 under full commitment.