

## 1 Lecture review

### 1.1 Diamond Dybvig

- Idiosyncratic preference shocks, incomplete markets
- Banks may be able to alleviate market incompleteness

### 1.2 Kiyotaki Moore

- Limited commitment  $\Rightarrow$  Collateral constraints
- Borrowing constraint dependent on prices  $\Rightarrow$  Re-allocation has feedback effects through prices

## 2 Idiosyncratic uncertainty

### 2.1 Static problem with iid preference shocks

Consider a two period economy with a perfectly storable consumption good. Agents are endowed with  $w_1$  of the good in period 1 and  $w_2$  in period 2. Agents are identical in period 1, and in period 2 are either “happy” with probability  $\pi$  or “sad” with probability  $1 - \pi$ . Period 2 shocks are only realized after agents have made their savings decision and are iid. Happy agents have preferences  $\log c_1 + \log(c_2 + \epsilon)$ , while sad agents have preferences  $\log c_1 + \log(c_2 - \epsilon)$  for some  $\epsilon > 0$ .

1. Setup and solve for the planner’s allocation
2. Setup and solve for the household’s problem in autarky. Are households better off in autarky compared to the planner’s allocation?
3. Suppose households can make enforceable contracts among each other in period 1. Setup and solve for the competitive equilibrium

### 2.2 OG problem with iid endowment shocks

Consider an OG economy with 2 period lived agents. Agents are endowed with  $w_1$  when they are young. When they are old, they are endowed with either  $w_2$  of the consumption good with probability  $\pi$  or 0 with probability  $1 - \pi$ . The endowment shocks are realized when they are old and are iid. Assume  $w_1 > w_2$ . Agents have preferences  $\log c_t^t + \beta \log c_{t+1}^t$ . The initial old are endowed with  $\pi w_2$  of the consumption good and  $\bar{M}$  units of valueless but perfectly storable currency. They have preference  $\beta \log c_1^0$ .

1. Setup and solve for the planner’s allocation
2. Setup and solve for the household’s problem
3. Solve for a steady state competitive equilibrium where the currency is valued
4. Compare allocations in (1) and (3). Are households better off with the allocations in (3)?

### 3 Aggregate uncertainty

#### 3.1 Static problem with iid and aggregate endowment shocks

Consider a two period economy with a perfectly storable consumption good. Agents are endowed with  $w_1$  in period 1 and  $zw_2$  in period 2. Here  $z, w_2$  are random variables that are realized in period 2.  $w_2$  is iid across agents, and takes on value  $w_h$  with probability  $\pi$  and  $w_l$  with probability  $1 - \pi$ .  $z$  is common for all agents, and takes on value  $z_h$  with probability  $\gamma$  and  $z_l$  with probability  $1 - \gamma$ . Agents have preferences  $\log c_1 + \beta \log c_2$ . Assume  $z_h > z_l, w_h > w_l$ , and  $w_1 \geq z_h w_h$ .

1. Setup and solve for the planner's allocation
2. Setup the household's problem in autarky. Without solving, can we say whether households are better off in autarky compared to the planner's allocation?
3. Suppose households can make enforceable contracts among each other in period 1. Setup and solve for the competitive equilibrium

### 4 IC/IR constraints

#### 4.1 Collateral constraint

Recall the collateral problem in class: 2 period model; 2 goods: non-storable consumption and storable housing; housing have relative prices  $q_0$  and  $q_1$  in period 0 and 1; households have income 0 and  $y_1$  in period 0 and 1; households can borrow  $b_1$  in period 0 to repay  $(1 + r)b_1$  in period 1. Assume that borrowers cannot commit to repay. But in the event of default, lenders can seize  $\kappa$  times the value of housing  $q_1 h_0$ .

In period 2, for a given value of  $b_1$  and  $h_0$ , when would households default on their debt? Argue how this leads to the collateral constraint

$$b_1 \leq \kappa \frac{q_1 h_0}{1 + r}$$

#### 4.2 Some information problems

1. A monopolist can choose the price  $p$  and quality  $q$  of a good sold. There are two types of consumers, each demanding a single unit of the good: a high type with preference  $u_h(q) - p$  and a low type with preference  $u_l(q) - p$ . The monopolist cannot distinguish between the consumer types, but sets a price and quality schedule  $\{(p_l, q_l), (p_h, q_h)\}$  such that high types pick  $(p_h, q_h)$  and low types pick  $(p_l, q_l)$ . What is the IC constraint on the price and quality schedule?
2. A government decides on the level of proportional income tax and the level of income transfer to households. Households have preferences  $\log c - l$ , where  $c$  is consumption and  $l$  is labour. There are two types of households, a high type that produces  $y = A_h l$  and a low type that produces  $y = A_l l$ . The government cannot distinguish between household types, but can set tax and transfer schedules  $\{(\tau_i, T_i)\}$  where  $\tau$  is the proportional income tax,  $T$  is the income transfer, and  $i \in \{l, h\}$ . Note that the after tax income to households are  $A_i(1 - \tau_i)l + T_i$ . What is the IC constraint on the tax and transfer schedules?