Econ712 - Problem set 1

1 Variations on the baseline OG

Consider an overlapping generations economy of 2-period-lived agents. There is a measure N of agents in each generation. New young agents enter the economy at each date $t \geq 1$. Half of the young agents are endowed with w_1 when young and 0 when old. The other half are endowed with 0 when young and w_2 when old. There is no savings technology. Agents order their consumption stream by $U(c_t^t, c_{t+1}^t) = \ln c_t^t + \ln c_{t+1}^t$. There is a measure N of inital old agents. Half of them are endowed with w_2 and the other half endowed with 0. Each old agent order their consumption by c_1^0.

1. In a given period t, set up and solve the problem of a planner that weighs everyone alive equally.
2. Suppose that agents who are alive can make fully enforcible contracts among themselves, and there exists a market for these contracts. What kind of contracts (ie who trades with whom) would be traded?
3. Set up and solve the problem of young agents born at time t, assuming they have access to the above market for contracts. Agents take prices as given.
4. Impose market clearing to solve for the prices and allocations. Compare the allocations to that in (1), and give intuition as to why they are similar/different.
5. Now suppose there is a government that has access to generation specific lumpsum taxes and transfers (they can only impose different tax/transfer across generations, whereas individuals of the same generation face the same tax/transfer). Shutting down the above contracting market, can the government implement the planner's optimal allocations? Can they implement the planner's allocation with the above contracting market?
6. Briefly discuss how incorporating population growth/shrinkage might change the above answers.

2 Setting up a problem

For the following, please set up (just set up, solve at your own leisure) the optimizing problem of the associated agent(s). Feel free to impose (and explicitly state) additional assumptions to help with modeling.

1. Time is discrete and runs forever. There are 2 goods in the economy, a capital good k and an all purpose good y. y can either be used for consumption c or for investment i to create more capital for the next period. There is a production technology that generates all purpose goods according to the function $y = k^\alpha$. Capital depreciates at rate δ, such that capital tomorrow is $k_{t+1} = (1-\delta)k_t + i_t$. The economy is endowed with k_0 at the start of period 0. The planner maximizes $\sum_{t=0}^{\infty} \beta^t u(c_t)$, choosing how much to consume and invest each period.
2. Consider an agent that lives for T periods with time seperable utility. She ranks consumption each period according to $u(C)$, and discounts future consumption geometrically at rate β. Each period she is endowed with w units of consumption good. She has access to a perfect storage technology, whereby 1 unit of good saved today will give her 1 unit of good tomorrow. Her problem is to optimize over savings and consumption for every period.

3. Consider an individual just entering the workforce, who will retire after N periods. Each period, his earnings y_t is the product of the number of hours worked h_t, the level of his human capital k_t, and the wage w_t: $y_t = h_t k_t w_t$. Assume that the wage is overtime. In each period, the worker can accumulate human capital by reducing the amount of hours worked. Specifically, $h_t = \phi(k_{t+1}/k_t)$. The worker seeks to maximize the present value of his earnings, at the discount rate $(1+r)^{-1}$, with a given k_0.

4. Consider a tree whose growth is given by the function h. That is, if k_t is the size of the tree in period t, then $k_{t+1} = h(k_t)$. Assume that once cut down, all of the wood must be sold at price p_t. Assume that the interest rate r is constant over time. Assume that it is costless to cut down the tree, and that the tree must be cut down by time T. The problem is to find the optimal time to cut down the tree to maximize the present discounted value, given an initial size k_0.

5. For a firm producing a new product, suppose that the marginal cost of production is constant in each period at c_t, but this marginal cost falls over time as a function of cumulative experience. Denote production and cumulative experience at time t by q_t and Q_t. Then $Q_0 = 0$ and $Q_{t+1} = Q_t + q_t$. Let marginal cost be related to experience by $c_t = \gamma(Q_t)$. The inverse demand is $p(q_t)$. The firm chooses production each period to maximize its present value of profits, at discount $(1+r)^{-1}$.