ECON 712A: Macroeconomic Theory - Problem Set 2

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Problem 1: Stock Market Reaction to FOMC Announcement[1]

Here we are interested in how the stock market may react to a future Federal Open Market Committee (FOMC) announcement of an increase in short term interest rates. To do so, consider the following simple perfect foresight model of stock price dynamics given by the following equation

$$p_t = \frac{d + p_{t+1}}{(1+r)} \tag{1}$$

where p_t is the price of a share at the beginning of period t before constant dividend d is paid out, and r is the short term risk free interest rate. The left hand side of (1) is the cost of buying a share while the right hand side is the benefit of buying the share (the owner receives a dividend and capital gain or loss from the sale of the share). Assume $r > 0$.

1. Solve for the steady state stock price $p^* = p_t = p_{t+1}$.
2. Assume the initial price, p_0, is given. The closed form solution to the first order linear difference equation (1) is:

$$p_t = \frac{d}{r} + \left(p_0 - \frac{d}{r}\right)(1+r)^t \tag{2}$$

   Explain how price evolves over time (i.e. what if $p_0 > p^*$, $p_0 < p^*$, $p_0 = p^*$) using both a phase diagram (i.e p_{t+1} against p_t) as well as a graph of p_t against time t. If the initial stock price is away from the steady state, does it converge or diverge from the steady state. Explain why.
3. [**Matlab**] Suppose the risk free rate is $r = 1\%$ and the stock pays constant dividend $d = 1$ per share per period. Open the Matlab code we provided. The code generates and plots the price dynamics given the first-order difference equation in part 2 given initial share price $p_0 = 100$ at time $t = 0$. Modify the code (i.e. simply replace 100) with three different initial prices which respectively are below, at, and above the steady state price level implied by these parameters to plot the price dynamics over 100 periods.
4. [**Analytical and Matlab**] Suppose the Federal Reserve announces at $t = 20$ to raise the federal funds rate from 1% to 2% at $t = 50$ and remain at the new level forever. Using (1) with $d = 1$, how does the price respond to the policy announcement and the interest rate change over time? Plot the price dynamics from $t = 0$ to $t = 99$. Please assume that we rule out rational bubbles (i.e., agents know and believe that prices will be at its fundamental value, the steady state, at t=50).

[1] Based on previous problem sets by Anton Babkin, Fu Tan, and Eirik Brandsås.

Problem 2: Neoclassical Growth Model

In the neoclassical growth model time is discrete and infinite. Households live forever. Each period, households consume c_t and save k_{t+1}. Production y_t is a function of the current capital level k_t. Capital depreciates at rate δ.

- Preference: $U(c) = \sum_{t=0}^{\infty} \beta^t u(c_t)$ where u is increasing, concave, and differentiable.
- Production technology: $y_t = f(k_t)$ where f is increasing, concave (DRS), and differentiable.
- Resource constraint: $c_t + k_{t+1} = y_t + (1-\delta)k_t$

Thus, the planners problem is:

$$\max_{(c_t, k_{t+1})_{t=0}^{\infty}} \sum_{t=0}^{\infty} \beta^t u(c_t)$$

$$\text{s.t. } c_t + k_{t+1} = f(k_t) + (1-\delta)k_t \ \forall t \tag{1}$$

1. Show that the planners problem implies the following Euler equation:

$$\beta u'(c_{t+1}) = \frac{u'(c_t)}{1-\delta+f'(k_{t+1})} \tag{2}$$

2. Let $f(k) = zk^\alpha$ and $u(c) = \log(c)$. Using the resource constraint and the Euler equation, solve for the steady state capital and consumption level $(\bar{k}, \bar{c})$ where $\bar{k} = k_t = k_{t+1}$ and $\bar{c} = c_t = c_{t+1}$.
3. Rewrite the equations (1) and (2) as the next period's level of capital and consumption as functions of today's level: $k_{t+1} = g(k_t, c_t)$ and $c_{t+1} = h(k_t, c_t)$.
4. On a phase diagram in (k_t, c_t) show how the system evolves after an unexpected positive productivity shock at t_0, $z' > z$. You don't need to plot lines precisely; pay attention to vector field (arrows), relative position of old and new steady states, directions of saddle paths, and system trajectory after the shock.
5. **[Matlab]** Let $\alpha = 0.3$, $\delta = 0.1$, and $\beta = 0.97$. Consider an unexpected change in productivity $z_t = z = 1$ for $t = 1, 2, ..., 4$ and $z_t = z' = 1.1$ for $t = 5, 6, ...$ using the "shooting method."

   (a) Compute $\bar{k}$ and $\bar{c}$ for z and $\bar{k}'$ and $\bar{c}'$ for z'. Use your formula from question 2.

   (b) Pick a guess for c_5. (Hint: What is the valid range for guesses of c_5?)

   (c) Solve for the trajectories of k_t and c_t for $t = 5, ..., T$ using the equations from question 3. Calculate the trajectory for many periods (say, $T = 10,000$).

   (d) If $k_T \neq \bar{k}'$ and $c_T \neq \bar{c}'$, update guess for c_5 and repeat (c). If $k_T = \bar{k}'$ and $c_T = \bar{c}'$, we know that we're converged to the steady state, so c_5 was the "correct" guess.

   (e) Plot c_t and k_t for $t = 1, ...20$. (Hint: $k_t = \bar{k}$ and $c_t = \bar{c}$ for $t = 1, ..., 4$).