Econ712 - Problem set 3

Variations on the OG model

Consider the following overlapping generations problem. In each period $t = 1, 2, 3, \ldots$ a new generation of 2 period lived households are born with mass N_t. Assume that $N_t = (1+n)N_{t-1}$, with $n \geq 0$. Each generation is endowed with w_1 in youth and w_2 in old age of a non-storable consumption good where $w_1 > w_2 \geq 0$. There is no commitment technology to enforce trades. The utility function of a household of generation $t \geq 1$ is

$$U\left(c_t^t, c_{t+1}^t\right) = \ln(c_t^t) + \ln(c_{t+1}^t)$$

where $\left(c_t^t, c_{t+1}^t\right)$ is consumption of a household of generation t in youth (i.e. in period t) and old age (i.e in period $t+1$).

There is a unit measure of initial old who are endowed with $\bar{M}_1 > 0$ units of fiat money and w_2 of the consumption good. Assume that the money supply increases over time, with $\bar{M}_t = (1+z)\bar{M}_{t-1}$ with $z \geq 0$. The increase in money supply is handed out each period to old agents in direct proportion to the amount of money that they chose when young. In other words, if a young agent chooses $M_{t+1}^t \geq 0$, they will receive $(1+z)M_{t+1}^t$ units of money when old. The preferences of the initial old are given by $U(c_1^0) = \ln(c_1^0)$ where c_1^0 is consumption by a household of the initial old.

1. State and solve the planner's problem under the assumption that every generation is weighted equally, so that the the objective of the social planner is to maximize $U(c_1^0) + \sum_{t=1}^{\infty} U\left(c_t^t, c_{t+1}^t\right)$. [1]

2. Let p_t be the price of consumption goods in terms of money at time t. Define a competitive equilibrium.

3. A stationary competitive equilibrium is defined as a competitive equilibrium with allocations that are constant through time (with possibly the exception of allocations in period $t = 1$). Solve for a stationary autarkic equilibrium, one where agents' optimal consumption choices are their own endowments.

4. Solve for a stationary monetary equilibrium (ie where money is valued) (Hint: Solve the agent's problem without the nonnegativity constraint on money, then verify that the non-negativity constraint on money is not binding in the equilibrium). What is the rate of return on money in the monetary equilibrium? Give intuition why the rate of return is at the level you find. Hints:

   (a) Solve for the initial old and generation t's problems given goods prices (p_t, p_{t+1}) for $t \geq 1$. That is, solve for $(c_t^t, c_{t+1}^t, M_{t+1}^t)$ given (p_t, p_{t+1}).

   (b) Denote $q_t := \frac{p_t}{p_{t+1}}$ and use goods market clearing with the optimal consumption functions $c_t^{t-1}(p_{t-1}, p_t), c_t^t(p_t, p_{t+1})$ derived above to get a first-order difference equation in q_t.

   (c) Solve for steady states $\bar{q}$. Show that one steady state of $\bar{q}$ corresponds to a steady state monetary equilibrium with inter-generational trade.

[1] This objective may not be well defined (i.e. add up to infinity), we can apply the "overtaking" criterion to determine optimality. The "overtaking criterion" states that an allocation $\{c_t^{t-1}, c_t^t\}_{t=1}^T$ overtakes $\{\widehat{c}_t^{t-1}, \widehat{c}_t^t\}_{t=1}^T$ if

$$\lim_{T\to\infty}\inf \left[U(c_1^0) + \sum_{t=1}^{T} U(c_t^t, c_{t+1}^t) - U(\widehat{c}_1^0) + \sum_{t=1}^{T} U(\widehat{c}_t^t, \widehat{c}_{t+1}^t)\right] > 0.$$

Since the finite sum is well defined, this sequence is well defined. Hence just go ahead and maximize away as usual.

5. Does the stationary monetary equilibrium Pareto dominate autarky?

6. For the stationary monetary equilibrium, briefly describe how the allocations change as we vary $\bar{M}_1$, z, and n. Provide some intuition explaining these results.

7. Now consider non-stationary equilibriums:

   (a) Derive the excess demand, and equations that characterize the offer curve for generation t

   (b) Graph the offer curve (roughly) along with the "market clearing" curve in excess demand space

   (c) For $w_2 > 0$, how many non-stationary equilibriums are there? Illustrate one of them (if one exists)

   (d) For $w_2 = 0$, how many non-stationary equilibriums are there? If your answer is different to the previous question, give intuition as to why