Econ712 - Problem set 5[1]

1 (Non-) Commitment in a black-box example with discrete choice sets

There is a continuum of identical households in the economy, able to choose some action $x \in \Xi$. Let the economy wide average (the aggregate) of these actions be X. The benevolent government chooses some action $y \in Y$. The payoff to a household is $u(x, X, y)$ - a function of there own action, the aggregate action, and the government action. Let the optimal choice of households, as a function of aggregates, be $f(X, y) := \arg\max_{x \in \Xi} u(x, X, y)$.

Define a competitive equilibrium as actions (x, X, y) such that

- Given y, x, households optimize, i.e. $x = f(X, y)$, and
- Household action is consistent with the aggregate, i.e. $\underbrace{f(X, y)}_{\text{HH action}} = \underbrace{X}_{\text{Aggregate action}}$.

Now for each y, define the aggregate action $h(y)$ to satisfy $f(h(y), y) = h(y)$. That is, $(h(y), h(y), y)$ is a competitive equilibrium.

Let $\Xi = \{x_H, x_L\}$, $Y = \{y_H, y_L\}$. In the following, subscripts $i, j, k \in \{H, L\}$. For the one-period economy, with $x_i = X_i$,the payoffs $u(x_i, X_i, y_j)$ are given by the following table:

	x_L	x_H
y_L	12^*	25
y_H	0	24^*

Table 1: One-period payoffs

Here the values $u(x_k, X_i, y_j)$ not reported are such that the outcomes with * are competitive equilibria. For example, $u(x_k, X_i, y_j) = -1$ for $k \neq i$ and $i = j$, and $u(x_k, X_i, y_j) = 30$ for $k \neq i$ and $i \neq j$. You should convince yourself that this is the case.

1. Find the Ramsey outcome, that is when the government has commitment/moves first. Find the outcome when the government cannot commit/moves second (in pure strategies). We will refer to this case as the *Nash equilibrium in pure strategies (NE).*

2. Suppose the economy is repeated 5 times. Can the Ramsey outcome be supported in any period?

[1] **Thesen problems draw extensively from Ljungqvist and Sargent's Recursive Macroeconomic Theory**

Now consider the expanded version of the previous economy. The payoffs $u(x_i, X_i, y_j)$ are given by the following table:

	x_{LL}	x_L	x_H
y_{LL}	2^*	6	10
y_L	1	12^*	25
y_H	-1	0	24^*

Table 2: One-period payoffs

3. What are the NEs? Suppose the economy is repeated 3 times, with agents discounting future utilites by $\beta = 0.9$. Can the Ramsey outcome be supported in any period?

2 Static taxation

Let there be a unit measure of households with preferences over leisure, (private) consumption, and public goods (l, c, g), defined by the utility

$$u(l, c, g) = \ln l + \ln(\alpha + c) + \ln(\alpha + g), \quad \alpha \in (0, 0.5)$$

Each household is endowed with 1 unit of time, which can be spent on leisure or labour. Production is linear in labour, i.e. the economy resource constraint is

$$\bar{l} + g + \bar{c} = 1$$

where $\bar{l}, \bar{c}$ are aggregate leisure and consumption. To provide the public good, the government can levy a flat proportional tax τ on labour. That is, $g = \tau(1 - l)$.

1. Set up and solve the Planner's problem.
2. Set up and solve for the Ramsey outcome. Hint: Gets quite messy - feel free to use an equation solver to find roots
3. Set up and solve for the NE outcome.
4. Comment on the differences between the above 3 outcomes, and the reason as to why they are different.
5. Suppose the economy is repeated for infinite periods, with discount factor $\beta < 1$. For high enough β, can the Ramsey outcome be sustained?