Econ 712A: Problem Set 6

1 Variation on Diamond OG Model

In this problem set, we analyze the dynamic equilibrium of a two period lived overlapping generations production economy with a constant unit measure of identical agents in each generation.

Each agent's preferences are given by the utility function

$$u(c_{t+1}^t, n_t^t) = \frac{1}{1-\gamma}(c_{t+1}^t)^{1-\gamma} - n_t^t$$

where c_{t+1}^t denotes period $t+1$ consumption of an agent born in period t (i.e. an old person), $n_t^t \in [0,1]$ denotes labor supplied in period t by an agent born in period t (i.e. a young person), and $\gamma \in (0,1)$.

Technology is given by the production function $f(k_t, n_t) = k_t^{1/2} n_t^{1/2}$ where k_t is total capital available to each producer in period t. Capital fully depreciates after production, so $k_{t+1} = i_t$ where i_t is investment chosen by the young in period t from their labor earnings. Young agents born in period t supply labor n_t^t at real wage w_t in order to buy capital k_{t+1} which they rent to firms in their second period of life at real gross return R_{t+1} to obtain funds $R_{t+1}k_{t+1}$ for purchasing consumption goods c_{t+1}^t.

Assume that γ is such that the constraints on labor supply never binds. Firm profits π_{t+1} are lump-sum transferred to the old.

1. Write down the optimization problem faced by a generation $t \geq 1$ agent. Solve for labor supply and investment decision rules.

2. Write down the optimization problem faced by a representative firm which rents labor at price w_t and capital at gross rate R_t to maximize real profits. Show that firm profits are zero.

3. Define a competitive equilibrium.

4. Show that a competitive equilibrium satisfies the following pair of first order difference equations if profit π_t is zero for all periods

$$\begin{aligned} k_{t+1} &= R_t k_t, \\ R_{t+1}^{1-\gamma} &= 4k_t^{\gamma} R_t^{1+\gamma}. \end{aligned}$$

5. Find the steady state competitive equilibria where $R^{CE} = R_t = R_{t+1}$ and $k^{CE} = k_t = k_{t+1}$.

6. Solve the planners problem.

   (a) Write down the planners problem and find the optimality conditions.

   (b) Find the steady state values of $c^* = c_{t+1}^t = c_{t+2}^{t+1}, k^* = k_t = k_{t+1}, n^* = n_t^t = n_{t+1}^{t+1}$.

(c) Compare the steady state of the planners solution to the steady state competitive equilibrium. Comment on why they are the same/different (recall the discussion of dynamic inefficiency in section 3.6 of the Diamond lecture notes).

7. Draw the phase diagram in (k_t, R_t) space. Include lines for $\Delta k = 0$ and $\Delta R = 0$ (which cross at the nontrivial steady state), the vector field arrows, and the approximate saddle path.

8. Suppose that the economy starts in a non-trivial steady state. Using your phase diagram, draw the dynamics of (k_t, R_t) after an unexpected negative shock to the capital stock (i.e. part of the capital accumulated by the young cohort is destroyed when they become old). Can we determine the changes in wages, labor, and/or consumption using the firm and HH optimality conditions? If so, how do such variables change?