

Econ712 - PS7

Consider a two period economy with an unit measure of households. In period 1, households are endowed with w_1 units of consumption. In period 2, each household either receives endowment w_h with idiosyncratic probability π , or w_l with probability $1 - \pi$. Assume $w_1 > w_h > w_l$. There is a perfect savings technology (s units of goods saved today return s units of goods tomorrow). Households have log preferences over consumption and do not discount future consumption: $U(c_1, c_2) = \log c_1 + \log c_2$. Households maximize their expected utility.

1 Planner's problem

1. Setup the planner's problem, assuming the planner weighs everyone equally
2. Solve for the planner's optimal allocation. Are the allocation realization dependent, i.e. do they differ depending on households' period 2 endowments?

2 Complete markets

Assume that households can trade an asset q that pays out only if their second period endowment is w_h . That is, if they buy one unit of the asset at price p in period 1, they get one unit of goods in period 2 iff their endowment is w_h .

1. Setup and solve for the household problem
2. Define and solve for the competitive equilibrium
3. Compare the allocation you just found to the planner's allocation. Give intuition as to why they are similar/different
4. How would your answers to this part change if, instead of the asset only paying out if second period endowment is w_h , the asset only pays out if second period endowment is w_l ?

3 Incomplete markets

Assume that households cannot make contracts with each other.

1. Setup and solve for the household problem
2. Compare the allocation you just found to the planner's allocation. Give intuition as to why they are similar/different
3. Suppose there was a government that could impose taxes/transfers to agents at different rates. That is, they could impose taxes/transfers T_h, T_l to agents with w_h, w_l respectively. Can the planner's allocation be implemented in this environment?

4 Private information

Now suppose that the government in (3.3) above is the sole agent with access to the savings technology (hence they could also impose taxes/transfers T_1 in period 1). However, the government cannot observe which households have w_h and which have w_l , and have to rely on households' statement of their income. That is, if a household declare that they have w_h (w_l) in period 2, they get T_h (T_l).

1. Suppose that households are really morally strict and can only tell the truth. Can the planner's allocation be implemented in this environment? If so derive the taxes/transfers scheme
2. Suppose that households can lie (ie declare w_h even when they have w_l)
 - (a) What are the incentive compatibility constraints in this case? (Hint: In words, the constraint states that telling the truth gives higher utility than lying)
 - (b) Does scheme in (4.1) satisfy the incentive compatibility constraints?