ECON 712B: Handout 1

Instructor: Job Boerma
TAs: Duong Dang and Alex von Hafften

ECON 712A Midterm[1]

1. State mathematically a benevolent social planner's problem who maximizes the expected utility of all agents in the economy across all states of the world subject to resource feasibility. *Big Hints: Given the environment, the planner's objective is* $\pi \log(c_1)+(1-\pi)\cdot\{pA\log(c_{2G})+(1-p)\log(c_{2B})\}$. *Further, note that resource feasibility must hold for each period* t *and for each aggregate state* ω.

   Answer: The planner's problem is to choose $W \in [0,1], S \in [0,1], (c_1, c_{2G}, c_{2B}) \in \mathbb{R}^3_+$ to solve

$$\max \pi \log(c_1) + (1-\pi)\left\{pA\log(c_{2G}) + (1-p)\log(c_{2B})\right\}$$

$$\text{subject to: } S + \pi c_1 = W \tag{1}$$

$$(1-\pi)c_{2G} = S + (1-W)R_G \tag{2}$$

$$(1-\pi)c_{2B} = S \tag{3}$$

3. State the incentive compatibility constraints that must be satisfied in an information constrained social planner's problem. Does the presence of private information change the resource feasibility constraints? If so, why. If not, why not?

   Answer: Incentive compatibility requires that the early consumers report to the planner that their type is early and the late consumers report their type as late.

   If early consumers report as late consumers they receive nothing in $t = 1$. If they do not consume at $t = 1$, they receive $\log(0) = -\infty$ utility. Given that c_1^* is strictly positive, the incentive compatibility constraint for the early consumer is trivially satisfied.

   The late consumers could report as early consumers, receive c_1^*, store it in the safe storage technology, and consume c_1^* in $t = 2$. The incentive compatibility constraint for the late consumer is:

$$pA\log(c_{2G}) + (1-p)\log(c_{2B}) \geq [pA + (1-p)]\log(c_1) \tag{4}$$

   If incentive compatibility is satisfied, then all types report truthfully and the planner knows the true mass of early and late consumers. Thus, resource feasibility is unchanged. If incentive compatibility is not satisfied, then all types report as early, so resource feasibility does change.

[1] Full answers on Dean's website: https://sites.google.com/a/wisc.edu/deancorbae/teaching.

Metric Spaces and Normed Vector Spaces[2]

Definition 1. *A* ***(real) vector space*** *X is a set of elements (vectors) together with two operations, addition and scalar multiplication. For any two vectors $x, y \in X$, addition gives vector $x + y \in X$, scalar multiplication gives a vector $\alpha x \in X$. These operations obey the usual algebraic laws, that is, for all $x, y, z \in X$, and $\alpha, \beta \in \mathbb{R}$ (with zero vector $\theta \in X$):*

1. $x + y = y + x$
2. $(x + y) + z = x + (y + z)$
3. $\alpha(x + y) = \alpha x + \alpha y$
4. $(\alpha + \beta)x = \alpha x + \beta x$
5. $(\alpha\beta)x = \alpha(\beta x)$
6. $x + \theta = x$
7. $0x = \theta$
8. $1x = x$

Definition 2. *A* ***metric space*** *is a set S, together with a metric (distance function) $\rho : S \times S \to \mathbb{R}$, such that for all $x, y, z \in S$.*

1. $\rho(x, y) \geq 0$ *with equality iff* $x = y$
2. $\rho(x, y) = \rho(y, x)$
3. $\rho(x, z) \leq \rho(x, y) + \rho(y, z)$

Definition 3. *A* ***normed vector space*** *is a vector space S, together with a norm $|| \cdot || : S \to \mathbb{R}$, such that for all $x, y \in S$ and $\alpha \in \mathbb{R}$:*

1. $||x|| > 0$ *with equality iff* $x = \theta$
2. $||\alpha x|| = |\alpha| \cdot ||x||$
3. $||x + y|| \geq ||x|| + ||y||$ *(triangle inequality)*

Definition 4. *A sequence $\{x_n\}_{n=0}^{\infty}$ in S* ***converges*** *to $x \in S$, if for each $\varepsilon > 0$, there exists H_ε such that $\forall n \geq N_\varepsilon$*

$$\rho(x_n, x) < \varepsilon$$

Definition 5. *A sequence $\{x_n\}_{n=0}^{\infty}$ in S is a* ***Cauchy sequence*** *(satisfies the* ***Cauchy criterion*** *if for each $\varepsilon > 0$, there exists N_ε such that $\forall n, m \geq N_\varepsilon$*

$$\rho(x_n, x_m) < \varepsilon$$

Definition 6. *A metric space (S, ρ) is* ***complete*** *if every Cauchy sequence in S converges to an element in S.*

Definition 7. *A* ***Banach space*** *is a complete normed vector space.*

Theorem 8. *Let $X \subseteq \mathbb{R}^l$, and let $C(X)$ be the set of bounded continuous functions $f : X \to \mathbb{R}$ with the sup norm $||f|| = \sup_{x \in X} |f(x)|$. Then $C(X)$ is a complete normed vector space.*[3]

[2] Based on section 3.1 of Stokey, Lucas, Prescott (1989).
[3] Theorem 3.1 from SLP.